A Random-Walk – based Approach to ARGESIM Comparison C19 'Pollution in Groundwater Flow'

F. Judex, TU Vienna; *efelo@fsmat.at*
G. Höfinger, ARCS Research Austria

Solution Approach: For this solution, a special Monte-Carlo method, a random walk (RW) in 2D is used. The pollutant is modelled as a finite number of particles with coordinates (x,y) and a concentration c. In each discrete time step Δt each particle is assigned a deterministic convective movement and a probabilistic dissipative movement:

$$x_{t+1} = x_t + u\Delta t + Z_x (2u\alpha\Delta t)^{\frac{1}{8}}, \quad y_{t+1} = y_t + Z_y (2u\alpha\Delta t)^{\frac{1}{8}}$$

Z_x and Z_y denote normally distributed random variables with $(\mu,\sigma)=(0,1)$. Each particle is assigned an appropriate amount of pollutant depending on the step size, on the source and on the depth of the aquifer.
Following [1], this approach converges under certain conditions to the solution of the transport equation.

Task a: Simulation of unaffected pollution spread.
Using a vector-oriented tool like MATLAB, a RW algorithm can be implemented in a few lines:

```
particles(time,:,1)=particles(time-1,:,1)+
          randn(1,perstep,1)*mult+xspeed;
particles(time,:,2)=particles(t-ime1,:,2)+
          randn(1,perstep,1)*mult;
```

There `particles` is a two-dimensional array for the coordinates, `mult` denotes the dissipative proportion and `xspeed` the convective fraction of the transport. For start, particles are fed into the system at (0,0). Calculation of the analytic solution (AS) is supported by MATLAB's standard function `erfc`.
The results show i) a qualitative coincidence between RW solution and AS solution for 50 days , a good quantitative coincidence at 100 days (Figure 1), and ii) no difference of RW solutions at 100 days and 150 days (a drawback of the RW method).

Task b: Pollution reduction by facilities.
Using MATLAB's `BesselK` function and using numeric integration, the concentration for each square of 1 x 1 m square is computed, so that an initial distribution of particles corresponding to the steady solution (SSL) is generated. Influence of the plants is modelled by varying the concentration (extension of the array of the particles).

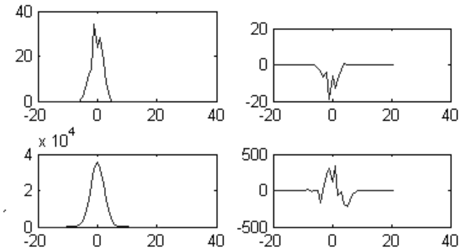

Figure 1: Concentration solution of RW (left) and absolute difference RW-SSL (right) for (50, *y*), *t* =50 (upper curves) and for (50, *y*), *t* = 100 (lower curves)

In each time step, particles inside the circles change their concentration by $\exp(\lambda\Delta t)$. Results (Figure 2) show the decreased concentration for the lines after the plants.

Due to the nature of RW, these solutions do not depend on the initial distribution (SSL), as all particles generated from SSL leave the given rectangle $-10 <= x <= 60$, $-20 <= y < =20$ before $t = 100$ days.

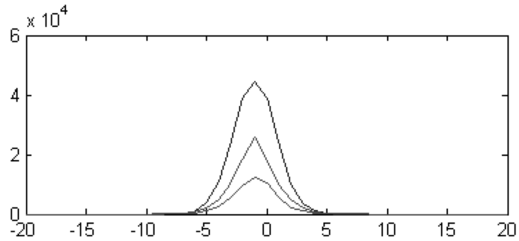

Figure 2: Decreasing concentration of pollution for (*30,y*), (*40,y*), and (*50,y*) at *t* =100 days

Task c: Controlled pollution reduction by facilities
To model the control of the plants, a boolean function switches on and off the reduction factor used before. Results (Figure 3) show, that pollution concentration during switched operation is 2.5 times higher than concentration during continuous operation. In switched operation, oscillating bebaviour can be observed (upper curve). The oscillations in continuous operation are due to the random nature of RW.

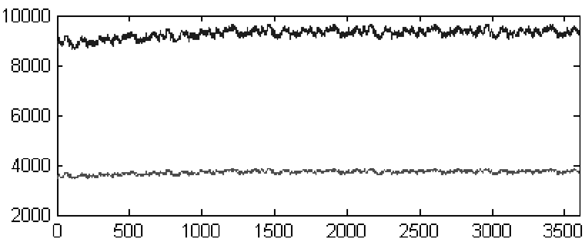

Figure 3: concentration at (50,0) for 0 to 3600 hours in mg, switched operation - upper curve, continuous operation - lower curve

[1] Kinzelbach, W., 1986. Groundwater Modelling - An Introduction with Sample Programs in BASIC, Elsevier Science Publ., Amsterdam