Pollution in Groundwater Flow - Definition of ARGESIM Comparison C19

G. Höfinger, ARC Seibersdorf Research,
Gerhard.Hoefinger@arcs.ac.at
F. Judex, F. Breitenecker, ARGESIM TU Vienna,
efelo@osiris.tuwien.ac.at

Since many years the demand for pure water is increasing, as well for human consumption as well as an ingredient in industrial processes. In many regions, the surface water available does not suffice, so more and more ground water has to be used. Exploring existing ground water bodies uncovers unfortunately many polluted areas, sometimes with unknown pollutant sources. In this exploration, data can only be gathered via wells, which is expensive and sometimes not possible. Therefore, modelling and simulation of a polluted groundwater body can help in various cases: determination of the pollution plume, localisation of the pollution source, planning of facilities for decrease of pollution, etc.

Basis for modelling is the transport equation, describing the pollution concentration, a PDE with constant or state-dependent parameters and more or less complex boundary conditions. Consequently different modelling approaches and solution techniques can be applied, from classical discretisation methods via FEM to alternatives techniques like cellular automata, and Monte-Carlo methods. In simple cases also approximating analytical solutions may exist. In reality, the choice of a modelling method or solution technique, resp. may also depend on the data available, and on the aim of the simulation.

This comparison investigates different modelling methods and different solution techniques for three tasks with increasing degree of difficulty.

PDE Model for pollution concentration

Basis for modelling is the transport equation, describing the concentration $c(t, x, y)$ of a pollutant in the saturated zone of a homogenous two-dimensional ground water body. A simplified version of the transport equation is

$$\frac{\partial c}{\partial t} - \frac{u}{R}\frac{\partial c}{\partial t} = \frac{\alpha_L u}{R}\frac{\partial^2 c}{\partial^2 x} + \frac{\alpha_T u}{R}\frac{\partial^2 c}{\partial^2 y} - \lambda c$$

Table 1 shows parameter values being typical for the slow flows under investigation.

pore velocity in x - direction	u	$u = 10^{-5}\ m/s$
dispersivity	$\alpha_T = \alpha_L$	0.05 m
retardation factor	λ	0
degradation	R	1
thickness of the saturated flow	h	10m
effective porous volume	n_e	0.25
input rate of pollutant mass	M	2 mg/s

Table 1: Parameter values for pollution spread

The effective porous volume n_e is the fraction of the water bearing stratum (aquifer) that really contains water, e.g. a cubic meter of material with an effective porous volume $n_e = 0.1$ can contain up to 100 liters – this maximum is reached in the saturated zone (considered in these investigation). The $h = 10$ meters of soil therefore represent 2.5 meters of water.

Assuming a steady source of pollutant M in (0, 0) on an infinite area allows the approximating solution for the parameters given in table 1:

$$c(x,y,t) = \frac{c_0}{4\sqrt{\pi\alpha}\sqrt{r}} e^{\frac{x-r}{2\alpha}} \operatorname{erfc}\left(\frac{r-ut}{\sqrt{2\alpha ut}}\right)$$

$$c_0 = \frac{M}{hn_e u}, \quad r = \sqrt{x^2 + y^2},$$

$$\operatorname{erfc}(x) = 1 - \operatorname{erf}(x)$$

Here it is assumed that the concentration is constant in the z - direction, and that retardation factor equals 1 (no retardation). For other parameter configurations, numerical techniques must be used in order to calculate a solution.

Task a: Simulation of unaffected pollution spread
Under simplified conditions, the concentration of pollution spreads from the source into x - direction like a plume (Figure 1). There exist a lot of approaches and numerical techniques for solving the transport equation. Aim of this task is to compare a numerical solution based on any technique with the approximate analytical solution for the simple case under investigation
For this purpose, an rectangular area with $-10 <= x <= 60$, $-20 <= y <= 20$ is to be chosen, with constant pollution source $M = 2.0$ mg/s in (0, 0) - other parameters see Table 1, with observation period of 150 days, and without pollution at starting.

Issue 44/45, December 2005

Issue 44/45, December 2005

Results should be compared with the analytical approximation at the line (50, y) at t = 50, t = 100, and t = 150 days (absolute values, differences).

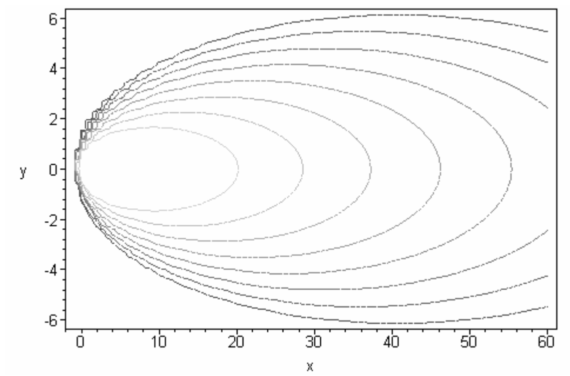

Figure 1: Pollution spreading from a pollution source, isolines

Task b: Pollution reduction by facilities

Main goal is to reduce or to eliminate the pollution. If the pollution source cannot be influenced directly, facilities can be set at certain locations reducing the pollution locally (wells with chemical substances, pumps blowing in oxygen for precipitation, etc.).
In the surrounding of such facilities locally elimination of the pollution takes place, reflected by an increase of the degradation parameter in the transport equation in a neighbourhood of the location.

The task is now, to investigate the influence of a facility with two plants starting with the steady state solution $c(x, y, inf)$. With K_0 being the modified Bessel function of second kind, and with c_0 and r as before, the steady state solution is given by

$$c(x,y,\infty)=\frac{c_0}{2}e^{\frac{x}{2\alpha}}K_0\left(\frac{r}{2\alpha}\right)$$

The facility consists of two plants situated at (40, 5) and (40, -5). Their influence on the pollutant is modelled by a change of the degradation parameter λ to the value of $\lambda = -10^{-6} ln10$ in a circle neighbourhood with a radius of 5 m. The implication of this change is that the concentration drops to 10 percent for a control volume travelling exactly 106 seconds in one of those neighbourhoods. Figure 2 sketches the situation.

The task is now, to model this setup appropriately and to simulate the system starting from the steady state solution. As result, a plot of the lines (30,y), (40,y), and (50, y), -20 <= y <= 20 for t = 100 days should be shown.

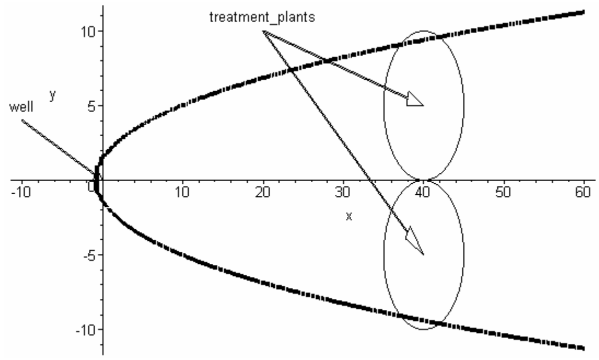

Figure 2: Boundary of pollution (thick line), influence areas of plants (thin circle lines)

Task c: Controlled pollution reduction by facilities

To minimize costs for operating the plants and to allow for maintenance, the hours of operation must be limited. A reasonable strategy lets the plants operate only during night and at weekend, so that maintenance can be done at regular working hours, and so that the cheaper electric energy during the night hours can be used.

This strategy can be modelled by a periodical change of the degradation parameter λ from $\lambda = -10^{-6} ln10$ (plants on) to $\lambda = 0$ (plants off).

Task is now, to model this strategy appropriately and to simulate the system starting from the steady state solution with the following strategy: facilities are active Monday to Friday from 0 to 8am and from 8pm to 12 pm, and at weekends around the clock.

As result, plots against time are now appropriate: plot the concentration at (50,0), i.e. $c(50,0,t)$ for 0 <= t <= 150 (days) for switched operation given above together with concentration for continuous operation (task b).

Solutions – Requirements and Structure

Solutions of any kind are appreciated, from FEM approaches to cellular automata, from discretisation methods to Monte Carlo methods, using simulators, libraries, packages or direct programming.
The solution should fit into one page SNE and consists of a description of the modelling approach, and of modelling and implementation details and results of the three tasks.

Solutions (to be sent to *sne@argesim.org*) may be accompanied by detailed PDF- or HTML – documentation and source code of the programs to be put on the ARGESIM server HTTP:/WWW.ARGESIM.ORG.