C12 Sphere's Collision – MATLAB

Analytical Simulation / Event-oriented Model

Simulator. MATLAB is a widely used software tool based on numerical vector and matrix manipulation.

Model. In this solution a MATLAB function `c12` programmed which calculates the time-vector of the impacts of the spheres, a discrete time-vector, analytically. Furthermore, kind of the impact (indicating the colliding spheres), the relative positions and the velocities of the spheres at the impact times are calculated and stored.

Input parameters are the initial velocity v_1 of the 1^{st} sphere, the restitution coefficient e, and a maximal number of impacts. Call and arguments are:

```
function
[t,Ort,v,k1,k1dot,AnzSt]=c12(x1dot,e,maxSt
)
t ... time vector of the impacts
Ort ... the relative position vector of
the spheres at the impact times
v ... the relative velocities yi, i=1..3
of the spheres at the impact times
k1, k1dot ... abs.pos. and vel. of sphere
1
AnzSt ... number of impacts in this run
```

Calling this function is equivalent to a simulation in the time domain: between the discrete time instants the movement of the spheres is linear and so the exact position of the spheres is at any time easily reconstructable.

Task a1) Simulation in time domain. Calling c12 with e=0.2 and reconstruction of the linear movement gives immediately the time courses of the differences of the sphere's movements (fig. 1)

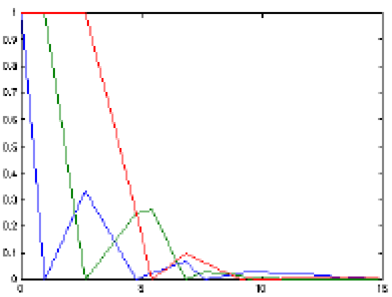

Figure 1: distance-time function y(t) simulated with e=0.2

Task a2) Final values of velocities. The final values of the absolut velocities of the four spheres for e=1 (elastic case) are: xdot=[0 0 0 1] and the final values of the absolut velocities of the four spheres for the quasi-plastic case with sufficiently equal velocities are for e=0.18, xdot=[0.25 0.25 0.25 0.25]

Task b) Variation of restitution coefficient. Calling c12 in a loop with varying restitution parameter e calculates the data for fig 2 (Number of impacts over e) and fig. 3 (final velocities over e).

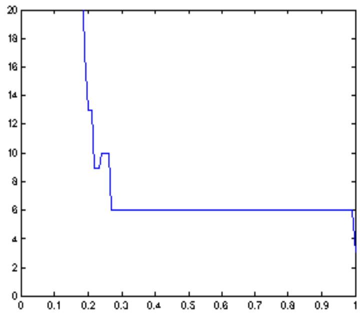

Fig. 2: Number of impacts n(e) over parameter e from 0.18 to 1

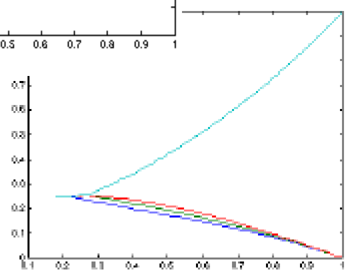

Fig. 3: Final velocities v_i over parameter e from 0.18 to 1

Task c1) Boundary value problem. The solution of the boundary value problem (Task c1)) has been realised with the Newton method. After four iterations the solution is found with an error smaller than 1e-6. The values for of e and for corresponding v4 are e = 0.5874011 v4 = 0.4999961.

Task c2) Statistical deviation of restitution. The function c12 is modified: the restitution parameter is not constant, it is normally distributed N(0.5, 0.05). A sample of 1000 simulation runs allows to calculate a statistical results for the final velocity v_4, displayed also in fig. 4. The numerical results are:

```
mean value = 0.42297, std.dev. = 0.02410
95%-confidence interval =[0.3734,0.4727]
```

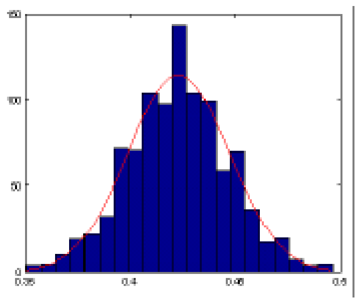

Fig. 4: Distribution function of final velocity v_4 for restitution parameter e out of N(0.5, 0.05).

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