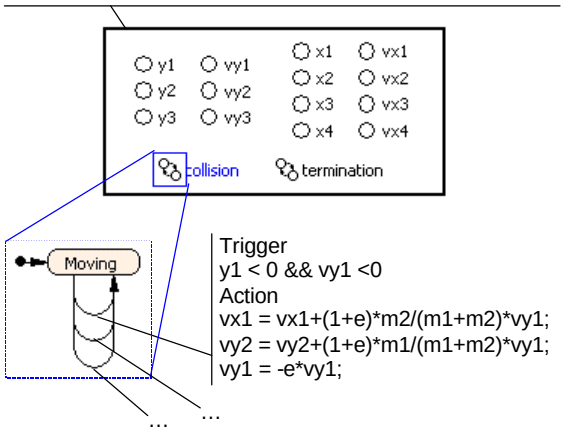
C12 Spheres' Collision – AnyLogic

Numerical simulation / Time-oriented Model

Simulator: AnyLogic (www.xjtek.com) is a general-purpose simulation environment for discrete, continuous and hybrid systems. It employs UML-RT structure diagrams for building hierarchical models in object-oriented way and *hybrid statecharts* for behaviour specification. The generated model is Java and can be extended with user's Java code. The simulation engine handles discrete events and dynamically changing sets of algebraic-differential equations. It automatically detects "change" (or "state") events.

Model: The system of colliding spheres is modelled as one AnyLogic object with several variables representing the positions of spheres (*x1*, *x2*, *x3*, *x4*), their relative distances (*y1*, *y2*, *y3*), and their absolute (*vx1*, *vx2*, *vx3*, *vx4*) and relative (*vy1*, *vy2*, *vy3*) velocities. The equations for motion of the spheres are associated with the object. The statechart has a single state and three transitions representing collisions of spheres. These transitions are triggered by "change events" – Boolean expressions over variables.

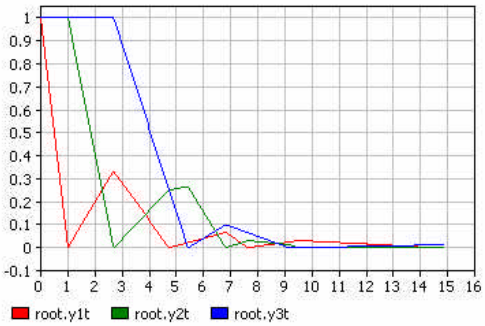
y1 = x2-x1-d	vx2 = vx1+vy1	d(x1)/dt = vx1
y2 = x3-x2-d	vx3 = vx2+vy2	d(x2)/dt = vx2
y3 = x4-x3-d	vx4 = vx3+vy3	d(x3)/dt = vx3
		d(x4)/dt = vx4



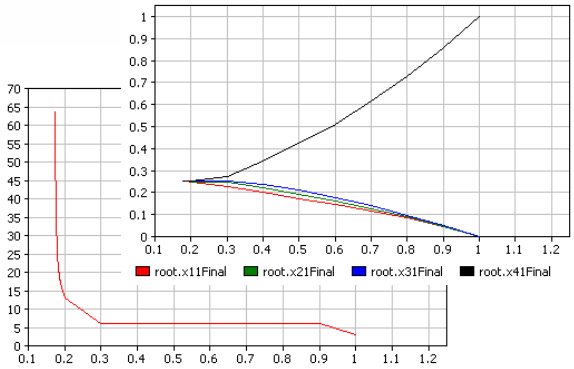
The hybrid statechart can be entered in AnyLogic model editor, and the simulation engine automatically detects change events, changes the working set of equations and readjusts the numerical methods.

Task a: Simulation of the System. a1: The following figure shows the distance-time functions *y1(t)*, *y2(t)* and *y3(t)* for $e = 0.2$, $d = 1$ and $a = 1$, $v0 = 1$ in time $0 \le t \le 15$ is shown on the diagram below.

Task a2. For the elastic case ($e=1$) the final velocities of the spheres are exactly *[0,0,0,1]*, for the quasi-plastic case ($e= 0.172$) *[0.25,0.25,0.25,0.25]*.



Task b: Variation of restitution coefficient: The number of collisions and final velocities as functions of the restitution coefficient presented in the picture below were obtained in a number of replications with varying parameter controlled by Java script.



Task c1: Boundary value problem. A binary search was implemented within AnyLogic environment as Java script. AnyLogic obtains the value of e = 0.5874011 with the precision set to 1e-7.

Task c2: Stochastic deviation of restitution. Using the built-in normal probability distribution for *e* AnyLogic has built the histogram shown below. The (built-in) statistical analysis gives the following results:

```
Mean 0.42317935397   Variance  4.54830611E-4
Min  0.35753660792   Deviation 0.02132675811
Max  0.50359726405   MeanConf  0.00132184616
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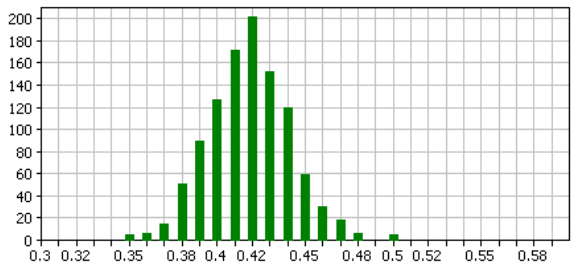

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