C12 Sphere's Collision – ACSL

Numerical-Analytical Simulation
Time-oriented Model

Simulator: ACSL is a continuous simulator, complemented by options permitting time- and state events, too. The comparison is suited to test the handling of a large number of state events and the numerical precision. Some partial tasks demand user routines in the TERMINAL section

Model. The distance/time functions $y_k(t)$, (k = 1, 2, 3) between two collisions can be described analytically because of the constant relative velocities. After an impact at t_i the distance functions between the spheres k and (k+1) have a simple form (linear slope).

For the absolute path/time function $x_j(t)$ of the spheres simple linear relations are used.

The remaining three functions $x_2(t)$, $x_3(t)$ and $x_4(t)$ are achieved by summing up $x_1(t)$ and $y_k(t)$.

Implementation**.** Now, besides the velocities according to the fitting conditions, also the accompanying distance co-ordinates $y_k(t_i)$, the time t_i have to make topical in the DISCRETE sections. The number of collisions n is incremented at every call of a DISCRETE section. Parts of the model description in ACSL's model description language:

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DERIVATIVE
x1 = x1d*(t-t0)+x10; y1 = y1d*(t-t1)+y10
SCHEDULE Collision12 .XN. y1
y2 = y2d*(t-t2)+y20;...END!of Derivative
DISCRETE Collision12
t0 = t; x10 = x1 ! path x1
t2 = t; y20 = y2
t1 = t; y10 = y1 ! distances y1, y2
x1d = x1d+(1.+e)*m2/(m1+m2)*y1d
y2d = y2d+(1.+e)*m1/(m1+m2)*y1d
y1d = -e*y1d; n = n+1
END ! of Collision12
```

Task a) Simulation in time domain / Final values of velocities. Just as the distance/time functions of task a1 (Fig. 1), the step functions of the velocities in case of spheres with little elasticity approach each other closely. Using e = 1, the final velocities of task a2 are $x_1 = x_2 = x_3 = 0$ and $x_4 = 1$. All four final velocities correspond for e = 0.18 in six decimal places with a value of 0.250000 (quasi inelastic/plastic case).

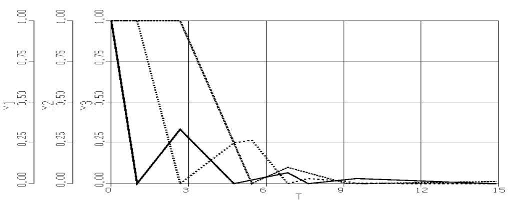

Fig. 1 Distance functions $y_1(t)$ solid, $y_2(t)$ dashed,

Task b) Variation of restitution coefficient. Fig. 2 shows the number of collisions n versus the decreasing restitution coefficient e with logarithmic scale. It achieves in case of double precision a maximum value of n = 1263 with e = 0.1715763. The final velocities of Fig. 3 point to the separate course of the last sphere.

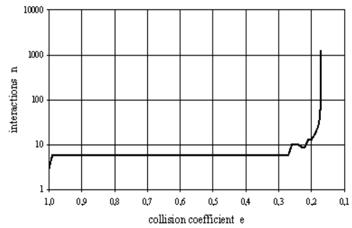

Fig. 2 Number of collisions $n(e)$

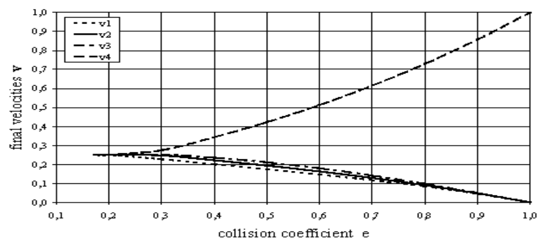

Fig. 3 Final velocities v(e), v_4 upper curve

Task c) Boundary value problem / Statistical deviation of restitution parameter. A *Newton* method, implemented in the TERMINAL section, using the allocations $f(x) := v_4 - v_0/2$ and $x := e$, iterates with ε = 1.0E-09 a value of e = 0.587401052 in 6 steps.

The statistic parameters with 1000 samples from the GAUSS(0.5, 0.05) function are mean value v_4 = 0.424898, standard deviation s = 0.041321 and confidence interval CONF{0.422337 ≤ μ ≤ 0.427459}

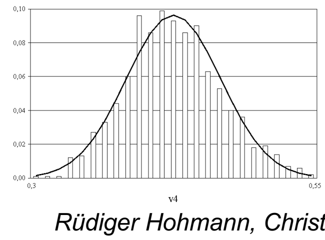

Fig. 4 Frequency comparison

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