Comparison 12 – ACSL

Hybrid Approach

The Simulator: ACSL is a simulator oriented on the continuous CSSL standard. In the problem observed hereunder, the features for treating state events (hybrid approach), the vector integration and the inclusion of user routines are relevant.

Model description: The distance/time and the velocity/time functions are calculated piecewise. The time periods are interrelated by fitting conditions. Every time two iteratively determined collision events form the integration boundaries for one interval. The ACSL model comprises the movement equations, the fitting conditions for the velocities in three DISCRETE sections as well as routines in the TERMINAL section for a variation in the collision coefficient, for iteration of the boundary conditions, and for generating an output file of stochastic values. All routines use iteration loops with returns to the INITIAL section. Because of the discontinuously varying velocities, the LOGD calls in the DISCRETE sections register variables both prior to the collision and after the model update with the ZZDERV(1) call. Simultaneously, the number of collisions is summed up. All computation was carried out with double precision. The conservation of the overall momentum can be used as a criterion for correctly representing the mechanics.

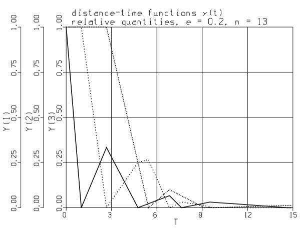

Figure 1: y_1 (solid), y_2 (dashed), y_3 (dotted)

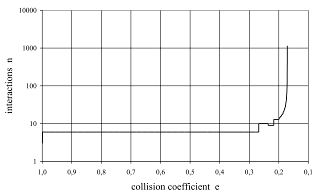

Figure 2: Number of collisions $n(e)$

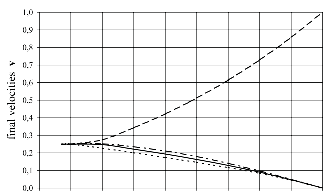

Figure 3: Final velocities v(e)

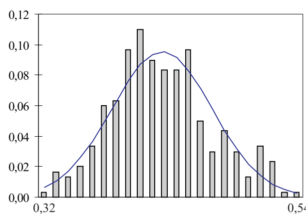

Figure 4: Frequency comparison

Results task a: The distance/time functions in Figure 1 of task a1 show that spheres of little elasticity approach each other closely. The final velocities in task a2 with e = 1 are $\dot{x}_1 = \dot{x}_2 = \dot{x}_3 = 0$ and $\dot{x}_4 = 1$. With e being 0.18 they equal $\dot{x}_1 = \dot{x}_2 = \dot{x}_3 = \dot{x}_4 = 0.250000$ practically (quasi plastic), which holds strictly for $e = 0$ only.

Results task b: Characteristic for task b1 are both the large number of interactions and their sharp rise with small impact values of $e < 0.2$ in the logarithmic scheme in Figure 2. Selected value pairs (e,n): (1;3), (0.5;6), (0.25;10), (0.22;9), (0.2;13), (0.18;25), (0.175;40), (0.172;115), (0.1715;567) and the maximum n achieved (0.171577;1151). The function curve does not rise monotonously all the time. The representation of the final velocities vector v in Figure 3 shows the con-tinuous transition from the elastic to the quasi plastic case of task b2.

Results task c: A Newton method with the allocations $f := v_4$-v_0 /2 and e being the independent variable iterates the solution of task c1 (e = 0.587401) as zero of f by BLOCK-IF instructions. The integer variable i controls the change between the calculation of derivation fp and a new collision coefficient e.

```
Newton Iteration - Selected Sequences
CONSTANT vend=0.5, eps=1E-6, de=0.01
INITIAL
i = 0
e = 0.9
1..CONTINUE
END ! of Initial
deltav = x4d-vend ! function f
b = (y1d .GE. 0.) .AND. (y2d .GE. 0.)&
.AND. (y3d .GE. 0.) ! termination criterion
TERMT(b)
TERMINAL
IF(i .EQ. 0) THEN ! write final values
OPEN(6, File='DATEN.DAT')
    WRITE(6, 2) e, x4d
    2..FORMAT(1X,'e:',F8.6,3X,'x4d:',F8.6)
    IF(ABS(deltav) .GT. eps*vend) THEN
    e_old = e ! new restitution coefficient
    f_old = deltav
    e = e_old+de ! derivative fp
    (f_prime)
    i = 1
    GOTO 1
    ENDIF
    ELSE
    f = deltav
    fp = (f-f_old)/de
    e = e_old-f_old/fp ! Newton method
    i = 0
    GOTO 1
    ENDIF
    CLOSE(UNIT=6)
    END ! of Terminal

    Iteration steps:
    e: 0.900000   x4d: 0.857375
    e: 0.637396   x4d: 0.548745
    e: 0.589207   x4d: 0.501708
    e: 0.587414   x4d: 0.500013
    e: 0.587401   x4d: 0.500000
```

A frequency comparison of the simulation data with the density of the MLE-fitted N (0,423;0,0417) distribution is depicted in Figure 4. N_j / n represents the fraction of the observed values of task c2 that would fall in the jth interval while p_j indicates the corresponding proportion of values sampled from the fitted distribution. For evaluating the *goodness of fit* a χ^2-test has been carried out with the data now grouped into $k = 20$ intervals. We could not reject our distribution hypothesis at the α = 0,10 level. Obtained statistical parameters are: $\overline{v}_4$ = 0.423, s = 0.0418, KONF{0.418 ≤ μ ≤ 0.428} with n = 300 samples from the GAUSS(m, s) function.

Rüdiger Hohmann, Christian Gotzel, Carsten Pöge, Institut für Simulation und Graphik, Otto-von-Guericke-Universität Magdeburg, PF 4120, D-39016 Magdeburg, hohmann@isg.cs.uni-magdeburg.de