



## Extended Fuzzy Control of a Two-Tank System - ARGESIM Benchmark C9R

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ARGESIM Benchmark C9R 'Extended Fuzzy Control of a Two-Tank System' is based on a nonlinear model for two coupled tanks, the first with inflow, the second with constant outflow. The inflow is to be controlled properly, so that the water level of the second tank holds a constant level. For control, the benchmark investigates fuzzy control within an overall digital control of the system. First, two fuzzy controllers with integral action are defined, to be analysed and tested with the tank system, both having sufficient compensation features. As alternative, a proportional fuzzy control is investigated, having a faster transient response, but no compensation. On modelling level, the benchmark investigates modelling techniques for the nonlinear system and methods and features for modelling discrete control and especially fuzzy control. On experiment level, control setup and transient responses are investigated. There, the first task requires calculation and display of the fuzzy control surfaces, independent on the model to be controlled. The second and third task deal with time domain analysis of the transient behaviour of the different fuzzy controllers under certain scenarios (changing setpoints for desired water level and disturbances). This benchmark extends and revises the ARGESIM Comparison C9 'Fuzzy Control of a Two-Tank System'.

### Introduction

The number of applications containing fuzzy components is still increasing. Modern simulation systems provide enhancements to implement fuzzy components in a convenient way. In SNE 17, July 1996, J. Goldynia has defined the first comparison, which should test the features of simulators with respect to fuzzy control. For this ARGESIM Comparison C9 'Fuzzy Control of a Two-Tank System', up to now 16 solutions have been sent in. Seven solutions make use of MATLAB/ Simulink, presenting different implementations. The other solutions showed implementations of fuzzy control in continuous and hybrid simulators, where the fuzzy module had to be programmed.

This revised extended definition improves the design of the fuzzy control and updates the tasks to be performed. The previous control design was not optimal. Main reason is a missing integral action in the control loop; consequently the revised definition designs the fuzzy control with integral action, to be compared with the a modified version of a proportional fuzzy control. The tasks to be performed are updated: instead of investigations with respect to weighted rules - being now a standard feature, a comparison of different types of fuzzy controllers is investigated. On experiment level, this new benchmark deals with control setup and transient responses are investigated. Task A requires calculation and display of the fuzzy control surfaces, independent on the two-tank - model. Task B and task C deal with time domain analysis of the transient response behaviour of the different fuzzy controllers under certain scenarios (changing setpoints for desired water level, and an isolated disturbance in the control).

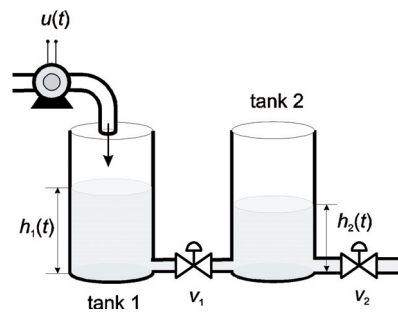


Figure 1: Schematic model of the two-tank system.

## 2 Dynamic Model for Two-Tank System

A two tank system (see Figure 1) in a specific configuration is characterised by the following nonlinear ODE set:

$$\begin{aligned}\dot{h}_1 &= c_1 u - f \\ \dot{h}_2 &= f - c_2 v_2 |h_2|^{0.5} \\ f &= c_3 v_1 \sqrt{|h_1 - h_2|} \text{sign}(h_1 - h_2)\end{aligned}$$

There  $h_1$  stands for the level in the first tank,  $h_2$  is the level in the second tank, the positions of the valves  $v_1$  and  $v_2$  are defined between 0 and 1. In our case the valve positions are  $v_1 = 0.4$ ,  $v_2 = 0.3$ . The process model includes characteristics of the liquid (laminar, turbulent, friction). The constants  $c_1$ ,  $c_2$  and  $c_3$  are:

$$\begin{aligned}c_1 &= 0.067 \text{ m} / \text{sV} \\ c_2 &= \begin{cases} 1.2 \cdot 0.0605 \text{ m}^{1/2} / \text{sV}, & h_2 < 0.16 \text{ m} \\ 1.0 \cdot 0.0605 \text{ m}^{1/2} / \text{sV}, & h_2 \geq 0.16 \text{ m} \end{cases} \\ c_3 &= 0.06624 \text{ m}^{1/2} / \text{sV}\end{aligned}$$

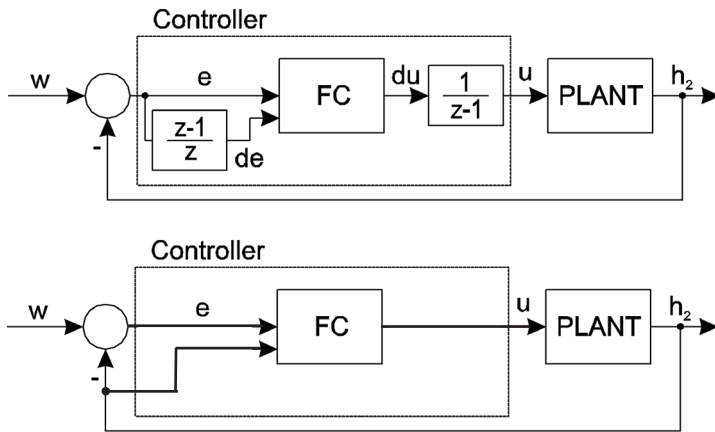


Figure 2: Closed-loop fuzzy control of liquid level  $h_2$ , IFC with integral action (upper picture), and PFC with pure proportional control (lower picture).

### 3 Fuzzy Control Design

For the two-tank system, two types of fuzzy control are designed. The task is to control the liquid level  $h_2$ . Both controls work with two inputs and one output. The first fuzzy control I-FC works with inputs  $e(k) = w - h_2(t_k) = w - h_2(k)$ ,  $w$  being the desired setpoint for  $h_2$ , and  $de(k) = e(k) - e(k-1)$ , and with integral action at the output; the second fuzzy control P-FC uses inputs  $e(k) = w - h_2(k)$  and  $h_2(k)$  without integral action at output. Both types are shown in Figure 2.

#### 3.1 Fuzzy Control with Integral Action - I-FC

The IFC fuzzy controller should be designed to have the capability to eliminate the steady-state control error and a suitable transient response. This means that the controller should have the integral control action. The controller should in general represent the following relation:

$$u(k+1) = u(k) + du(k)$$

$$du(k) = f(e(k), de(k))$$

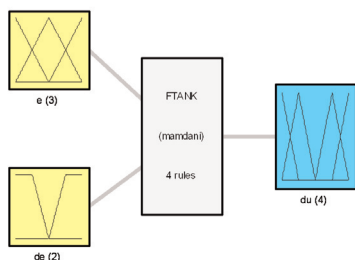


Figure 3: Structure of fuzzy controller.

The structure of the fuzzy controller should be the following: two input variables, divided in two and three membership functions and one output variable which is divided into four membership functions. The structure is shown in Figure 3. Two fuzzy controllers, I-FC1 and I-FC2 are designed. They should be implemented as discrete systems operating at one second sampling time.

The type of the fuzzy controllers should be *Mamdani*, what means that the output should be defined by membership function, the operator of the *intersection* between fuzzy sets is defined as the *product* of membership values and for the *union* operator between fuzzy sets the *sum* of membership values is used. The *defuzzification* method

which has to be used to calculate the crisp output is defined as *center of gravity* and for *implication* the *product* is used. Membership functions for  $e$  and  $de$  are defined in Figure 4a and 4b, membership functions of the output variable  $du$  in Figure 4c.

The membership functions for variables  $e$  and  $de$  are the same in the case of controller I-FC1 and I-FC2, the difference between both fuzzy controllers is in segmentation and form of the output fuzzy variable, which should be in I-FC1 in a form of *triangle membership functions* as shown in Figure 4c and in the case of I-FC2 in the form of *singleton membership functions* which are shown in Figure 4d (approximation of singleton functions). The controller should have integral action (I-FC) to eliminate the control error in steady state and to have the transient response as fast as possible (short settling time).

The *if-then* rules of both controllers should be the same and they should be given as four rules base in the following way, where the number in brackets gives the weighting of the rules:

1. **If** (e is low) **then** (du is open-fast) (1)
2. **If** (e is high) **then** (du is close-fast) (1)
3. **If** (e is good) **and** (de is falling) **then** (du is open-slow) (1)
4. **If** (e is good) **and** (de is rising) **then** (du is close-slow) (1)

#### 3.2 Proportional Fuzzy Controller - P-FC

Alternatively a fuzzy controller with a proportional fuzzy control P-FC is designed. This controller should be in general represented by the following relation:

$$u(k) = f(e(k), h_2(k))$$



This functional dependency between control error  $e(k)$ , the output variable  $h_2(k)$  and the control variable  $u(k)$  enables compensation of nonlinearity because the value of the control signal depends on the current operating point of the system. The control is suitable in the case of small, neglectable uncertainties and disturbances. The same operators for the basic fuzzy operations should be used as in the case of I-FC1 and I-FC2. The *if-then* rules of the P-FC controller are given as five rules base in the following way:

1. **If** (e is low) **then** (u is P5) (1)
2. **If** (e is high) **then** (u is P1) (1)
3. **If** (e is good) **and** (h<sub>2</sub> is high) **then** (u is P4) (1)
4. **If** (e is good) **and** (h<sub>2</sub> is low) **then** (u is P2) (1)
5. **If** (e is good) **and** (h<sub>2</sub> is middle) **then** (u is P3) (1)

The membership functions of the input variables for the P-FC controller are defined in Figure 5a for variable  $e$  and in Figure 5b for variable  $h_2$ . The output membership functions are defined in Figure 5c.

### 3 Tasks - Experiments

The ‘new’ benchmarks require for a solution a short description of the simulator (S), a detailed description of modelling (M), description and results of experiments (tasks A, B, C), and a short résumé (R) for classification.

**Modelling.** For documentation of modelling and implementation, please:

- describe the features for fuzzy control in your simulator or the interface to an interfaced fuzzy tool,
- model the controllers I-FC1 and I-FC2 (and P-FC) by means of features of the simulator or an appropriate additional tool linked to the simulator,
- and give a rough model description of the I-FC controllers and of the overall model, and indicate model changes for replacing the I-FC controllers by a proportional fuzzy controller P-FC.

Furthermore, describe, how the two-tank system is modelled (textually, graphically, library-based), and how system model, fuzzy control model, and additional discrete control blocks are composed to the overall model.

**A-Task: Fuzzy Controller Surfaces.** Compute and visualise the 3-dimensional characteristic surface of the fuzzy controllers I-FC1 and I-FC2. Define  $e$  in interval  $[-30, 30]$  on  $x$ -axis,  $de$  in the interval  $[-0.05, 0.05]$  on  $y$ -axis, and  $du$  on the  $z$ -axis.

If your system does not support singletons directly, you may use any kind of emulation. Indicate, whether your system evaluates directly the fuzzy control algorithm, or whether it generates a control surface, which is interpolated.

**B-Task: Transient Response.** Simulate the whole fuzzy control system using I-FC1 and I-FC2 alternatively for changing offset values and control disturbance. The reference signal and the disturbance should have the following profile:

- $w = 0.30$  m in the interval of first 20000 seconds,
- in the next 20000 seconds the reference equals  $w = 0.60$  m,
- and in the interval of the last 20.000 seconds it should be  $w = 0.40$  m;
- additionally, at time instant 50.000 seconds a process input disturbance occurs:  $u_z = 0.1$  V.

Plot  $h_2$ ,  $h_1$  and  $u$  versus time, for I-FC1 and I-FC2. The simulation time is equal 60.000 seconds. Compare IFC1 and IFC2 by indicating differences in quality of control and behaviour caused by disturbance.

**C-Task: Comparison with Proportional Fuzzy Control.** This task should compare the the integral-type fuzzy controllers I-FC1 and I-FC2 with the proportional fuzzy controller P-FC.

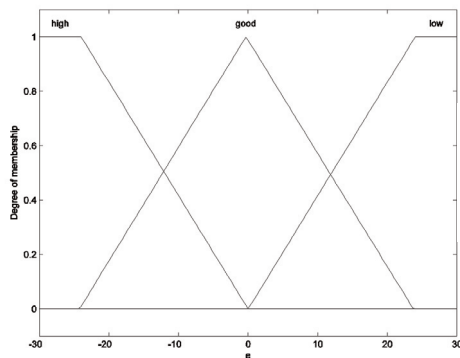
The following tasks should be performed:

- Comment the necessary changes of the model.
- Compute and visualise the 3-dimensional characteristic (surface) of the fuzzy controller P-FC ( $e$  in interval  $[-30, 30]$  on the  $x$ -axis,  $h_2$  in the interval  $[0, 100]$  on the  $y$ -axis, and  $u$  on the  $z$ -axis).
- Show time domain simulation results for the P-FC controller with same scenario as in case of I-FC controllers and discuss qualitative control behaviour and reaction on disturbance.

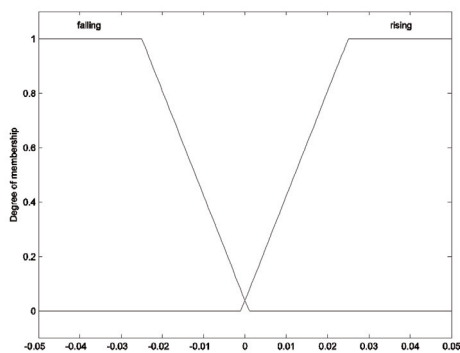
### 4 Solutions - Results

The expected results show certain advantages and disadvantages of controllers I-FC and P-FC. The transient response of P-FC can be very fast, but the control error can not be eliminated. On the other hand, I-FC can reject the control error, but its action is relatively slow. These fuzzy control designs can be combined to a controller of type PI-FC, having advantages of both types.

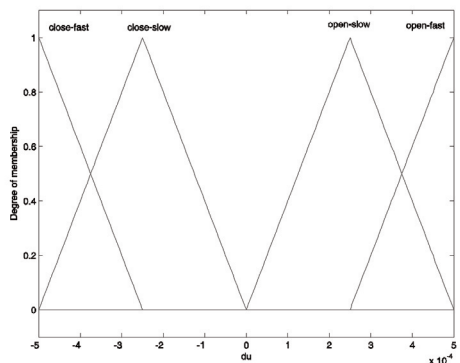
Solutions should fit into two pages SNE. Templates for text (.doc or .rtf) may be downloaded from [WW.ARGESIM.ORG](http://WW.ARGESIM.ORG), menue *SNE* or at [WWW.ASIM-GL.ORG](http://WWW.ASIM-GL.ORG), menu *International - SNE*.



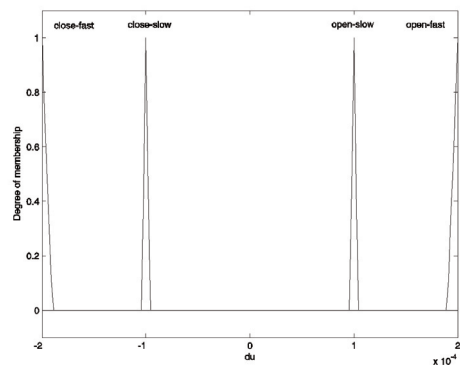
a: Membership function for  $e$ , I-FC1 and I-FC2



b: Membership function for  $de$ , I-FC1 and I-FC2



c: Membership function for  $du$ , I-FC1



d: Membership function for  $du$ , I-FC2

Figures must be also provided separately as file, any format, also formulas in case authors are not using the .doc or .rtf - template (e.g. .pdf). Furthermore, all model files (and batch files, etc., if any) must be sent in properly documented style - they will be provided at the web, so that readers can download and can make use of them.

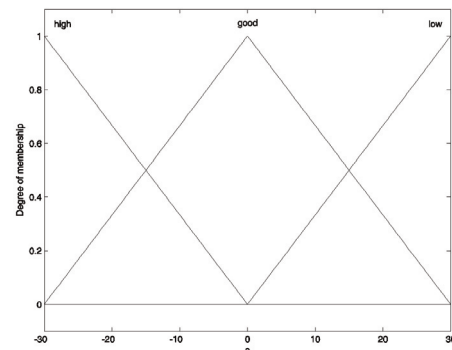
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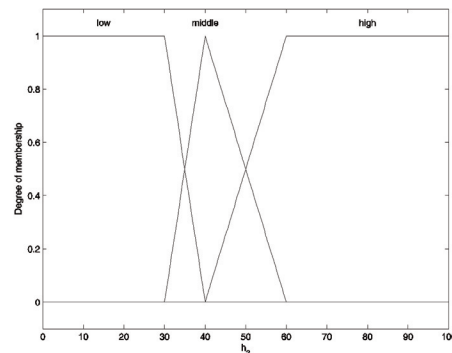
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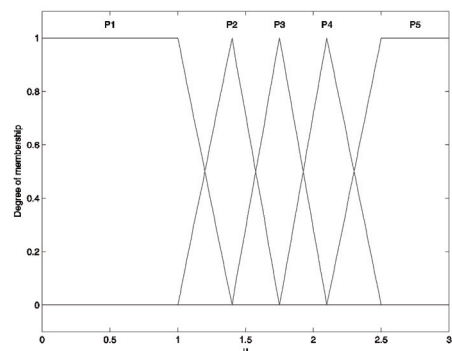
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a: Membership functions for  $e$ , P-FC



b: Membership functions for  $h_2$ , P-FC



c: Membership functions for  $u$ , P-FC

Figure 4: Membership functions for I-FC1 and I-FC2; a, b: membership functions for  $e$  and  $de$  (I-FC1, I-FC2); c, d: output membership functions for I-FC1 and I-FC2 ( $du$ ).

Figure 5: Membership functions for P-FC; a, b: membership functions for  $e$  and  $de$ ; c: output membership functions ( $du$ ).