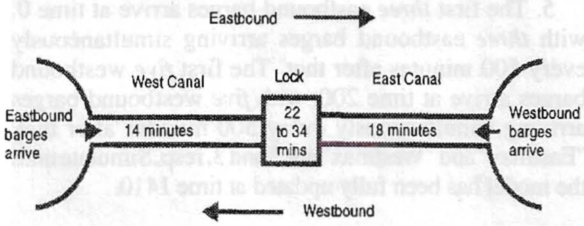
# Comparison 8: Canal-and-Lock System

The objectives in this discrete comparison are to investigate features for modeling complex logic, to validate logic through use of deterministic data, and to check for variance-reduction capabilities.

### The Canal-and-Lock System

The following figure shows a Canal-and-Lock System used by barges moving from one waterway to another. The system is composed of a West Canal, a Lock, and an East Canal.




Eastbound and westbound barges arrive at the Canal-and-Lock System with interarrival times specified in datasets that follow. Barge travel times in the West Canal and East Canal are 14 and 18 minutes, respectively (deterministic as a first approximation). (Barges might spend more time than this in the Canals, however, because of potential delay at the Lock.) After having arrived at the Lock and being next in line to use it, from 22 to 34 minutes are spent by a barge before it has finished using the Lock. Details are given below.

Because the Canals are narrow, barges can move through the system in only one direction at a time. The policy for operating the system is to let up to "Eastmax" barges proceed through the system in the *eastbound* direction, and then let up to "Westmax" barges proceed through the system in the *westbound* direction, and so on. It is not always the case, however, that during a west-to-east cycle there are at least "Eastmax" barges that want to move eastbound through the system, or that during a east-to-west cycle there are at least "Westmax" barges that want to move westbound through the system. The following considerations apply:

1. If there are *no* barges in the system when a barge arrives, then that barge initiates a barge-movement cycle in the direction in which it is headed.

2. A barge-movement cycle ends either when the specified per-cycle maximum allowable number of barges has moved in the direction in question (this is called a "full" cycle), or when the maximum has not yet been reached but there are no more barges waiting to move in that direction (this is called a "partially full" cycle).

3. When a barge-movement cycle ends and there are barges waiting to move in the opposite direction, then a cycle is initiated in that opposite direction.

4. When a cycle ends and *no* barges are waiting to move in the *opposite* direction but there *are* additional barges waiting to move in the *current* direction, then a cycle in the current direction is *freshly initiated.* (In a freshly initiated cycle, the count of barges that have moved through the system during the freshly-initiated cycle is set to zero and then builds up again, potentially reaching the "Eastmax" or "Westmax" value before the direction of movement is potentially changed.)

5. When a cycle ends and there are *no* barges waiting to move in *either* direction, then eventually another barge arrives and consideration 1 above is in effect.

The Lock itself can hold only one barge at a time. Service order for Lock use is first-come, first-served (for the barges currently *in* the Lock-and-Canal System).

Now consider the particulars of barge movement through the Lock. Assume (arbitrarily) that the water level in the West Canal is higher than in the East Canal, so that the Lock is used to *lower* eastbound traffic and to *raise* westbound traffic. Assume the current direction of barge movement is *eastbound*, that an eastbound barge has just moved out of the Lock into the East Canal, and that the next eastbound barge has already arrived at the Lock in the West Canal and is waiting to use the Lock. Then to move this next eastbound barge through the Lock, these steps take place:

1. The water level of the Lock (which has no barge in it) is raised to that of the West Canal. This takes 12 minutes.
2. The next eastbound barge moves into the Lock. This takes 5 minutes.
3. The water level in the Lock is lowered to that of the East Canal. This takes 12 minutes.
4. The barge moves out of the Lock into the East Canal. This takes 5 minutes.

The net effect of steps 1, 2, 3 and 4 is that it takes 34 minutes in this *maximum time* case for this next eastbound barge to *pass through the Lock.*

Now assume in the above scenario that the next eastbound barge hasn't yet arrived at the Lock in the West Canal when its predecessor finishes moving out of the Lock into the East Canal. Assume further that the next eastbound barge doesn't arrive at the Lock until 12 or more minutes *after* its predecessor has moved out of the Lock into the East Canal. Then the unoccupied Lock's water level will already have been raised to that of the West Canal by the time the next eastbound barge reaches the Lock. The net effect is that it will only take 22 minutes (corresponding to steps 2, 3 and 4 above) in this *minimum time* case for this next eastbound barge to pass through the Lock. (Note the assumption that the Lock-and-Canal System is intelligent enough to change or start to change the water level in the Lock, if necessary, in anticipation of the level at which the next barge will arrive at the Lock.)

Finally assume that in the above scenario, the next eastbound barge arrives at the Lock after its predecessor has moved out of the Lock, but before the re-filling of the Lock has been completed. Then this next barge must wait for the re-filling to be completed. For example, if the predecessor leaves the Lock at time 500 and its successor arrives at the Lock at time 508, then only 8 minutes of Lock-refilling time have gone by and the successor will have to wait 4 minutes before the refilling of the Lock will be completed. As a result, this barge spends a total of 26 minutes (4 remaining minutes for step 1, then the time for steps 2, 3 and 4) passing through the Lock.

The reasoning set forth above is valid by symmetry for westbound traffic.

**Task a) The Modeling Assignment**

Model the Canal-and-Lock System. Design the model so that various alternative settings of "Eastmax" and "Westmax" and barge interarrival times can be experimented with easily. The model should measure average barge transit times, in minutes, on both a segregated (by direction of traffic movement) and pooled (for traffic movement in both directions) basis. (Barge transit time is defined as the elapsed time between arrival of a barge *outside* the Canal-and-Lock System at one end and its eventual departure from the system at the other end.) What fraction of the model is devoted to expressing the logical complexities of the system?

**Task b) Model Validation with Deterministic Data**

Several deterministic datasets are provided below, each designed to test various aspects of your model's behavior. Use your model to perform single simulations for the datasets. Compare your model's results with those given in the Appendix. All correctly built models should produce identical results for these datasets.

1. The first eastbound barge arrives at time 0, with one eastbound barge arriving every 108 minutes after that. The first westbound barge arrives at time 54, with one westbound barge arriving every 108 minutes after that. "Eastmax" and "Westmax" are each 1. Simulate until the model has been fully updated at time 1458.

2. The first *two* eastbound barges arrive at time 0, with *two* eastbound barges arriving simultaneously every 210 minutes after that. The first *three* westbound barges arrive at time 88, with *three* westbound barges arriving simultaneously every 210 minutes after that. "Eastmax" and "Westmax" are 2 and 3, resp. Simulate until the model has been fully updated at time 1470.

3. The first eastbound barge arrives at time 0, with one eastbound barge arriving every 54 minutes after that. No westbound barges ever arrive. "Eastmax" and "Westmax" are each 1. Simulate until the model has been fully updated at time 1458.

4. The first westbound barge arrives at time 0, with one westbound barge arriving every 100 minutes after that. No eastbound barges ever arrive. "Eastmax" and "Westmax" are each 1. Simulate until the model has been fully updated at time 1500.

5. The first *three* eastbound barges arrive at time 0, with *three* eastbound barges arriving simultaneously every 500 minutes after that. The first *five* westbound barges arrive at time 200, with *five* westbound barges arriving simultaneously every 500 minutes after that. "Eastmax" and "Westmax" are 2 and 3, resp. Simulate until the model has been fully updated at time 1410.

**Task c) Variance Reduction Experiments**

The following datasets call for the use of two variance reduction techniques to form interval estimates of expected (or differences in expected) pooled barge transit time. For purposes of estimating the effectiveness of these techniques in the context of this problem, the forming of interval estimates without the use of the variance-reduction methodology is also specified.

1. Consider the case in which barge interarrival times are exponentially distributed, with expected interarrival times of 75 minutes both for eastbound and westbound barges and with "Eastmax" and "Westmax" each set to 5. Form a 90% confidence interval for expected barge transit time on a pooled basis. Base the confidence interval on 100 independent replications, each of duration 14,400 simulated minutes. (Each replication corresponds to 10 simulated 24-hour days.) Assume there are no barges in the system when each replication begins, and that the arrival time of the *first* eastbound barge equals the value sampled from the uniform distribution 15 ± 15 minutes. Assume similarly that the arrival time of the *first* westbound barge equals the value sampled from the uniform distribution 10 ± 10 minutes. (Subsequent barge interarrival times are then exponentially distributed, as indicated above.) Also assume that transient conditions, if any, are negligible.

2. To provide a more balanced set of insights, repeat activity 1 two more times, but change the starting points of the random number generators.

3. Repeat activities 1 and 2, but now use the Antithetic Random Variates (ARVs) variance-reduction metho-

dology, basing the 90% confidence intervals on 50 negatively correlated pairs of replications. (The total number of replications here equals those in activities 1 and 2, so the same amount of simulation effort is involved.) How much narrower are the resulting confidence intervals than the ones produced in activities 1 and 2?

4. For probabilistic dataset 1, it has been proposed that the expected pooled barge transit time could be decreased by setting "Eastmax" and "Westmax" each to 6. Proponents of this approach argue that it takes time waiting for the last barge in a cycle to leave the Canal-and-Lock System before the direction of traffic is potentially changed, and that it is better to write off this time overhead across 6-barge batches than 5-barge batches (whenever a full cycle occurs). Opponents of this approach point out that while the sixth barge in a 6-barge batch is moving through the system, all barges waiting to move through the system in the opposite direction are forced to wait that much longer. Which of these two alternative policies leads to the smaller estimated expected pooled barge transit time? Investigate the answer to this question by forming a 90% confidence interval for the *difference* in the pooled average barge transit times for the two alternatives (subtracting 6-barge pooled average transit times from 5-barge times). Base the confidence interval on 50 pairs of independent replications, each of duration 14,400 simulated minutes. Consider the null hypothesis that the expected waiting time for 5-barge batches is less than or equal to that for 6-barge batches. With the probability of a Type I error set at 0.05 (which corresponds to your 90% confidence interval), can you reject this null hypothesis?

5. To provide a more balanced set of insights, repeat activity 4 two more times, but change the starting points of the random number generators.

6. Repeat activities 4 and 5, but now use the Common Random Numbers (CRNs) variance-reduction methodology, basing the 90% confidence interval on 50 positively correlated replication pairs. (Note that the total number of replications here is equal to those in activities 4 and 5, so the same amount of simulation effort is involved.) How much narrower are the resulting confidence intervals than the ones produced in activities 4 and 5? With the probability of a Type I error set at 0.05, can you reject the null hypothesis that the expected waiting time for 5-barge batches is less than or equal to that for 6-barge batches?

## Appendix: Discussion of Deterministic Datasets

In formulating the complex logic behavior the following considerations may be helpful: (1) If an arriving barge detects there are no other barges in the system, it must initiate a cycle. (2) If an arriving barge detects there is an ongoing cycle in its direction of movement, it must enter its first canal if the cycle is not yet full and must wait outside that canal if the cycle is already full. (3) When a barge reaches the Lock, it must determine how long it has to wait before the water in the Lock is at the barge's level. (4) When a barge leaves its second canal, a check must be made to see if the barge is the last in the ongoing cycle; if so, a signal must be set that triggers initiation of a cycle in the opposite direction if appropriate, or initiation of another cycle in its own direction if appropriate, or puts the system into a state of suspension, waiting for the next arrival of a barge.

Dataset 1: Each eastbound cycle (full at one barge) is followed immediately by a westbound cycle (full at one barge), and vice versa. All barges pass through the system in the minimum feasible time of 54 minutes, so the average transit time is 54 minutes. When the simulation ends, the 14th eastbound barge has just left the East Canal, and the 14th westbound barge is just entering the East Canal.

Dataset 2: Each eastbound cycle (full at two barges) is followed immediately by a westbound cycle (full at three barges), and vice versa. Transit times for the first and second eastbound barges in a cycle are 54 and 88 minutes; and for the first, second and third westbound barges in a cycle are 54, 88, and 122 minutes. When the simulation ends at time 1470, the 21st westbound barge has just left the West Canal (7 westbound cycles have been completed, with an average transit time of 88 minutes), and the 15th and 16th eastbound barges are just entering the West Canal (7 eastbound cycles have been completed, with an average transit time of 71 minutes, and the 8th is just beginning).

Dataset 3: Each eastbound cycle (full at one barge) is immediately followed by another eastbound cycle (full at one barge). There are no westbound barges. The transit time for each barge is 54 minutes, so the average transit time is 54 minutes. When the simulation ends at time 1458, the 27th eastbound barge has just left the East Canal, and the 28th eastbound barge is just entering the West Canal.

Dataset 4: Each westbound cycle (full at one barge) is followed 46 minutes later by another westbound cycle (full at one barge). There are no eastbound barges. The transit time for each barge is 54 minutes, so the average transit time is 54 minutes. When the simulation ends at time 1500, the 16th westbound barge is just entering the East Canal.

Dataset 5: Each pair of consecutive eastbound cycles (the first cycle in the pair is full at two barges; the second cycle is partially full at one barge) is followed by a pair of consecutive westbound cycles (the first cycle in the pair is full at three barges; the second cycle is partially full at two barges), and vice versa. Transit times for the first and second eastbound barges in a full cycle are 54 and 88 minutes; and for the only eastbound barge in a partially full cycle is 142 minutes. Transit times for the first, second and third westbound barges in a full cycle are 54, 88, and 122 minutes; and for the first and second westbound barges in a partially full cycle are 176 and 210 minutes. When the simulation ends, the 15th westbound barge has just left the West Canal, and no barges are waiting to enter the Lock-and-Canal System at either end. Average transit time for the 9 eastbound barges that have moved through the system is 94.67 minutes; and for the 15 westbound barges that have moved through the system is 130 minutes.

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