A mixed Analytical – Numerical Solution of ARGESIM Comparison "C7-Constrained Pendulum" with Maple

Stefan Pawlik, F. Breitenecker, TU Vienna
pawlikb@fsmat.at

Simulator: Maple is a widely used Computer Algebra System, which also contains many numerical tools. Therefore it is a good choice if parts of the problem can be done symbolically.

Model: A change of the state space was done to simplify the task. This approach uses the tangential velocity instead of the angular velocity, which leads to a system of two simpler differential equations:

```
> diffsys:={diff(phi(t),t)=psi(t)/l(phi(t)),
  diff(psi(t),t)= -g*sin(phi(t))-d/m*psi(t)}:
> d:=0.2: g:=9.81: m:=1.02: phi_pin:=-Pi/12:
> l:=p->piecewise
    (p>=phi_pin, 1, p<phi_pin, 0.3):
> solu1:=dsolve(diffsys union
    {phi(0)=Pi/6, psi(0)=0},{phi(t),psi(t)},
     type=numeric, method=rkf45,
     output=listprocedure):
```

In then above equations the change of the pendulum length was achieved with the use of the `piecewise` function.

Task a –Simulation of the System: The numerical algorithm used was a RKF45 (without state event handling) which gives the following graphs for hitting the pin in case of task a-1. In case of task a-2 parameters in the `piecewise` – function have to be changed.

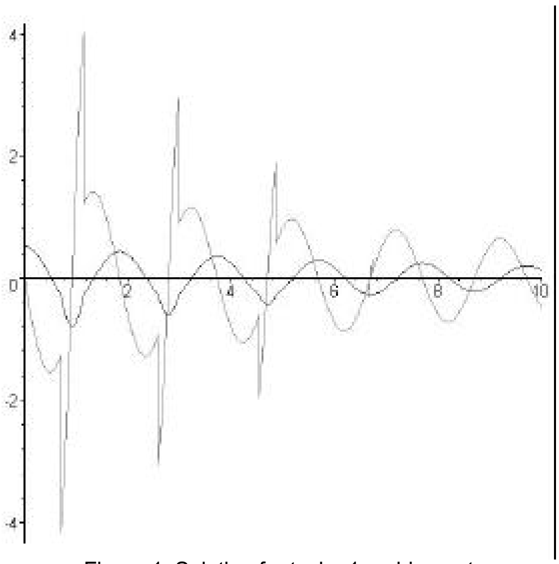

Figure 1: Solution for task a1 – phi over t

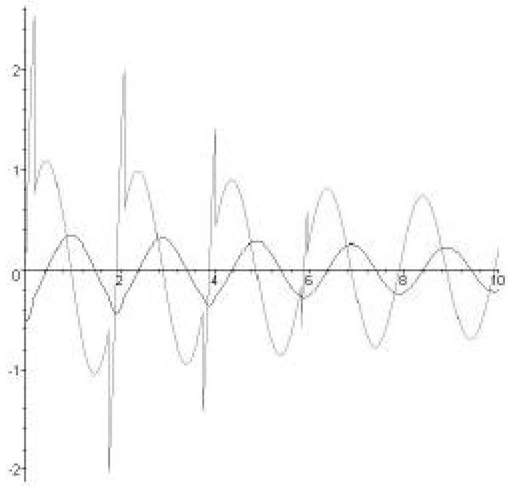

Figure 2: Solution for task a2 – phi over t

Task b - Comparison of nonlinear and linear model. Maple can calculate the analytical solution of the linear problem by symbolic computation:

```
>diffequlin:={m*l(phi(t))*diff(phi(t),t,t)=
        -m*g*phi(t)-d*l(phi(t))*diff(phi(t),t)}
```

The exact solution can be calculated till the pendulum hits the pin, then the next part of the solution can be calculated exactly with the new starting conditions which are the conditions at the time the pendulum hits the pin (by inverting the solution getting time as function of the angle), and so on (results in figure 3)

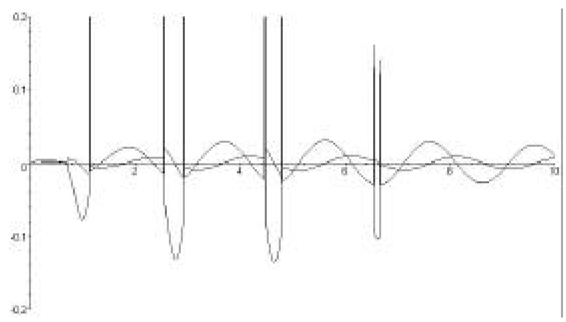

Figure 3: Difference of angle and angular velocity

Task c – Boundary Value Problem. This Problem can be seen as an initial value problem with inverse time. The initial values are angle = $-\pi/2$ and angular velocity = 0. Then only the time till the angle is $\pi/6$ has to be calculated and we are done.

C7 Classification: Analytical/Numerical Approach
Simulator: Maple 8.01