Three Structural Different Modelling Approaches to ARGESIM Comparison C7 'Constrained Pendulum' using the Modelica-Simulator MOSILAB

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Simulator: *MOSILAB* (*MO*deling and *SI*mulation *LAB*oratory) is a simulator developed by Fraunhofer-Institutes FIRST, IIS/EAS, ISE, IBP, IWU and IPK within the research project GENSIM. It is a generic simulation tool for modeling and simulation of complex multidisciplinary technical systems. The simulation environment supports the procedures modeling, simulation and post processing. The model description in MOSILAB is done in the MODELICA standard. Additional features to assure high flexibility during modeling the concept of structural dynamics is implemented. This is done by extending the Modelica standard with state charts, controlling dynamic models. The model description language resulting is called MOSILA. Moreover, simulator coupling with standard tools (e.g. MATLAB / Simulink, FEMLAB) is realised.

Modelling. The motion of the constrained pendulum is usually defined with φ and $\dot{\varphi}$ as states. But using the tangential velocity $v = l\dot{\varphi}$ instead of angular velocity, has the benefit, that only the length of the pendulum has a discrete change in case of hitting or leaving the pin:

$$\dot{\varphi} = \varphi\frac{v}{l}, \quad \dot{v} = -g\sin\varphi - \frac{d}{m}v$$

Standard Modelica approach. In this approach only standard MODELICA code is used. It is defined in the MOSILAB equation layer as implicit law (it is non necessary to transform to an explicit state space):

```
equation /*pendulum*/
  v = l1*der(phi); vdot = der(v);
  mass*vdot/l1 + mass*g*sin(phi) + damping*v = 0;
```

The state event, which appears every time when the rope of the pendulum hits or 'leaves' the pin, is modelled in an `algorithm` section with if (or when) - conditions:

```
algorithm
  if (phi<=phipin) then length:=ls; end if;
  if (phi>phipin) then length:=l1; end if;
```

This section defines length allocation of the constrained pendulum for all tasks. MOSILAB handles the if-clause (when-clause) by means of an state event finder.

MOSILAB state chart approach. This approach makes use of an additional feature of MOSILAB, modelling of discrete elements by state charts, which may be used instead of if- or when- clauses, with much higher flexibility and readability in case of complex conditions. Boolean variables define the status of the system and are managed by the statechart:

```
event Boolean lengthen(start=false),
        shorten(start = false);
equation
  lengthen=(phi>phipin); shorten=(phi<=phipin);
.. here /*pendulum*/ -equations .....
statechart
  state LengthSwitch extends State;
   State Short,Long,Initial(isInitial=true);
   transition Initial -> Long end transition;
   transition Long -> Short event shorten action
        length := ls;
   end transition;
   transition Short -> Long event lengthen action
        length := l1;
   end transition; end LengthSwitch;
```

From the modelling point of view, this description is equivalent to the description with if-clauses. The MOSILAB translator clearly generates there an implementation with different internal equations. MOSILAB's simulator performs simulation by handling the state event within the integration over the simulation horizon.

Hybrid model decomposition approach. MOSILAB's state chart construct is not not only a good alternative to if- or when - clauses within one model, it offers also the possibility to switch between structural different models. This very powerful feature allows any kind of hybrid composition of models with different state spaces and also of different type (from ODEs to PDEs, etc.). In case of the constrained pendulum, we decompose the system into two different models: `Short` pendulum model, and `Long` pendulum model, controlled by a state chart (Fig. 1).

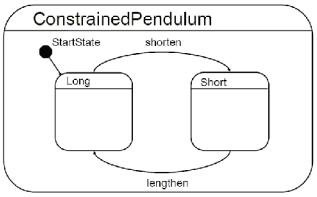

Figure 1: MOSILAB implementation with two different models controlled by a state chart

The model description defines now first the two pendulum models, and then the event as before:

```
model Long
equation
  mass*vdot/l1 + mass*g*sin(phi) + damping*v = 0;
end Long;
model Short
equation
  mass*vdot/ls + mass*g*sin(phi) + damping*v = 0;
end Short;
 event discrete Boolean lengthen(start=true),
                        shorten(start = false);
equation
 lengthen = (phi>phipin);shorten=(phi<=phipin);
```

The following state chart creates first instances of both pendulum models during the initial state (`new`). The transitions organise the switching between the pendulums (`remove`, `add`). The `connect` statements are used for mapping local states to global state variables:

```
statechart
 state ChangePendulum extends State;
 State Short,Long,startState(isInitial=true);
 transition startState -> Long action
  L:=new Long(); K:=new Short(); add(L);
 end transition;
 transition Long->Short event shorten action
  disconnect ….; remove(L); add(K); connect …
 end transition;
 transition Short -> Long event lengthen action
  disconnect …; remove(K); add(L); connect ……
 end transition; end ChangePendulum;
```

A - Task: Simulation of the System: Simulations were performed with all three modelling approaches, giving same results. The approach with state charts allow an easier handling of the different initial conditions for this task, because no additional if-clauses are necessary. Furthermore, simple extensions of the state chart would allow also arbitrary initial conditions. For simulation, MOSILAB's IDA-DASSL solver was used, Figure 2 and Figure 3 show results.

B - Task: Comparison of Linear and Nonlinear Model. The linearised model is implemented in the same way as the the nonlinear model, just substituting $\sin\varphi$ with φ. To calculate the difference of the results both state equations can be put into one model, or one can use state chart to couple nonlinear and linear model in parallel (results shown in Figure 3):

```
model nonlinear
equation
  mass*vdot/ls + mass*g*sin(phi) + damping*v = 0;
model linear
equation
  mass*vdot/ls + mass*g*phi + damping*v = 0;
equation
  difference = phi - phiLin
statechart
 state combine extends State
  State run, init(isInitial=true)
  transition init -> run action
    L:= new linear(phiLin=L.phi);
    NL:= new nonlinear(phi=NL.phi);
    add(L); add(NL);
end transition; end combine;
```

C - Task: Boundary Value Problem: One way to solve this problem is optimisation, using simulator coupling with MATLAB/Simulink. A simpler way is to transform the problem to an initial value problem by integrating equation backwards in time. The simulation is stopped event-controlled, when the desired angle is reached. The solution is the angular velocity at $t = 0$, which is approximately 2.185.

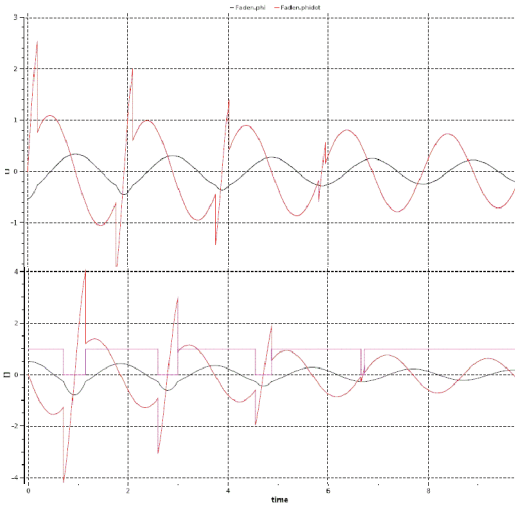

Figure 5: Angle φ, angle velocity and switching variable for different intial values of the nonlinear pendulum.

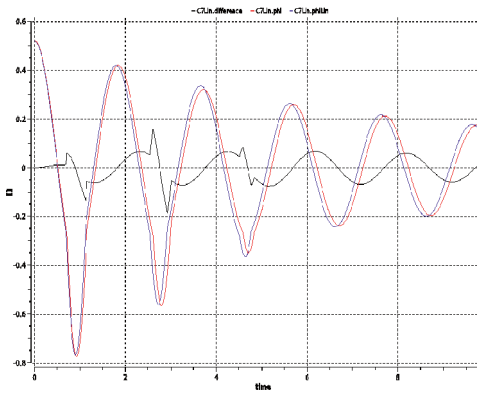

Figure 6: Angles φ for nonlinear and linear model with angle difference

Résumé. For system modelling and modelling of the state events, classical constructs from Modelica were used in a first approach; two other approaches model the state space changes by state charts (available in MOSILAB 2.0) controlling submodels (Tasks **M**). Scenarios with arbitrary different initial values are easily modelled by state charts for the initial phase; standard Modelica if-clauses may become here complex (Task **A**). Model comparison is done by simulating the models in parallel, for comparison modelling also state chart control is used (Task **B**). The boundary value problem (Task **C**) is simply solved by reversing time.

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