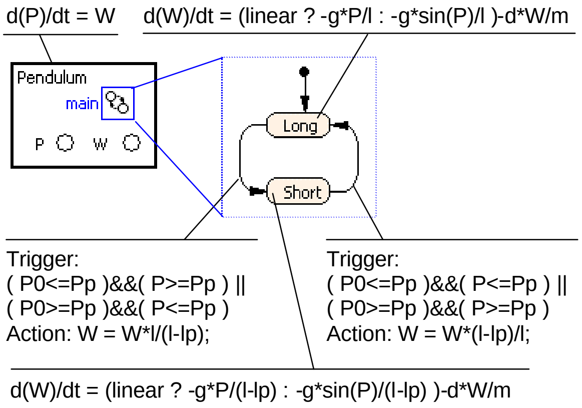
C7 Constrained Pendulum – AnyLogic

Hybrid Modelling Approach - Model Level

Simulator: AnyLogic (www.xjtek.com) is a general-purpose simulation environment for discrete, continuous and hybrid systems. It employs UML-RT structure diagrams for building hierarchical models in object-oriented way and *hybrid statecharts* for behaviour specification. The generated model is Java and can be extended with user's Java code. The simulation engine handles discrete events and dynamically changing sets of algebraic-differential equations. It automatically detects "change" (or "state") events. Debugging and visualization facilities are present.

Model: The constrained pendulum – essentially a hybrid system – is modelled by a hybrid statechart in a very natural and concise way, see figure below. The object Pendulum has two variables: angle *P* and angular velocity *W*. Obviously, the equation *d(P)/dt = W* is always valid; therefore it is associated with the whole object. The statechart has two states corresponding to different rotation points. Each state has an associated set of equations, which are active only while the control resides in the state.
Transitions between the states are triggered by "change events" – Boolean expressions over variables. The two expressions specify the conditions of hitting and leaving the pin.



d(P)/dt = W   d(W)/dt = (linear ? -g*P/l : -g*sin(P)/l)-d*W/m

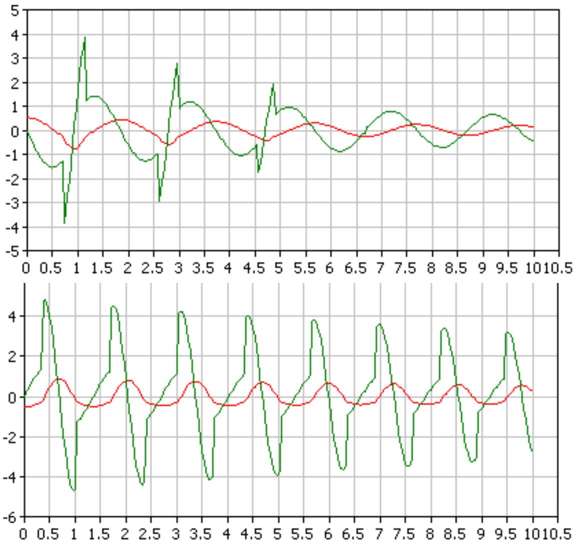
Trigger:
(P0<=Pp)&&(P>=Pp) ||
(P0>=Pp)&&(P<=Pp)
Action: W = W*l/(l-lp);

Trigger:
(P0<=Pp)&&(P<=Pp) ||
(P0>=Pp)&&(P>=Pp)
Action: W = W*(l-lp)/l;

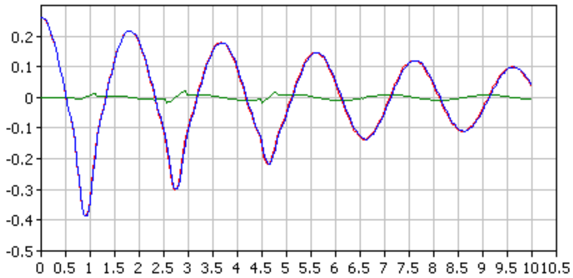
d(W)/dt = (linear ? -g*P/(l-lp) : -g*sin(P)/(l-lp))-d*W/m

The hybrid statechart can be entered in AnyLogic model editor, and the simulation engine automatically detects change events, changes the working set of equations and readjusts the numerical methods.

Task a: Simulation of the System. The initial values of the variables are parameters of the object Pendulum, so the user can modify them in AnyLogic model viewer (debugging and experimenting environment). The plots showing the angle and the angular velocity are for the cases **a-i** and **a-ii** are given below.



Task b: Comparison linear / nonlinear model: Depending on the Boolean parameter *linear*, the model can represent either linearised or non-linearised case (conditional right-hand side of equations in the state machine). To compare the cases two instances of Pendulum object with different values of *linear* were created and run in parallel, results below.



Task c: Boundary value problem was solved in two ways. A binary search was implemented within AnyLogic environment as Java script, controlling the simulation. With precision set to 1e-7, and the initial boundaries of the velocity are [-3,-1], AnyLogic calculates the value -2.184699.

However, the problem can also be solved in one simulation run by using the target conditions (*P*=-p/2 and *W*=0) as initial values and inverting the sign of the damping factor. The velocity at the first cross of the original angle p/6 is the desired result. A "watchdog" statechart with a change event *P*==p/6 was used to detect the crossing. This method gives the value of -2.1846992117869215.

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