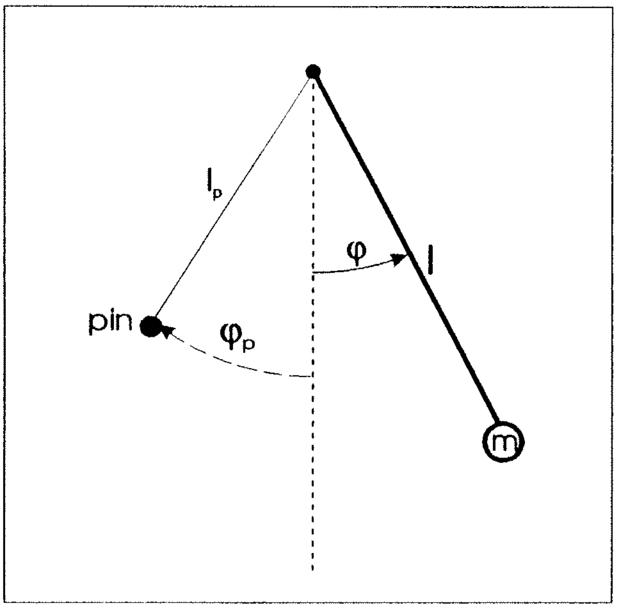
Comparison 7: Constrained Pendulum

This comparison tests features of simulation languages regarding state events, comparison of models, and parameter variation. The system under investigation is a constrained pendulum.

The motion of the pendulum is given by the equation

$$m\,l\,\ddot{\varphi} = -m\,g\,\sin\varphi - d\,l\,\dot{\varphi}$$

where φ denotes the angle measured in radians counter-clockwise from the vertical position. The parameters *m* and *l* characterize the pendulum with mass *m* and length *l*, *d* is a damping factor.



If the pendulum is swinging, it may hit a pin positioned at angle φ_p with distance l_p from the point of suspension. In this case the pendulum swings on with the position of the pin as the point of rotation and the shortened length $l_s = l - l_p$.

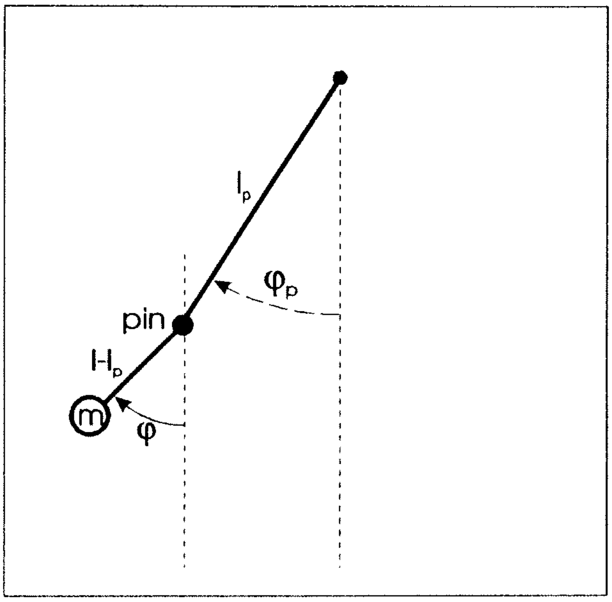
Note that the angular velocity $\dot{\varphi}$ is defined now with respect to the new point of rotation; therefore the angular velocity $\dot{\varphi}$ is changed at position φ_p from $\dot{\varphi}$ to $\dot{\varphi}\frac{l}{l_s}$.

The above equations remain valid.

If the pendulum swings back and passes φ_p, the pendulum behaves as before with length *l*, and the angular velocity $\dot{\varphi}$ is changed at φ_p from $\dot{\varphi}$ to $\dot{\varphi}\frac{l_s}{l}$, and so on.

General parameters for the following tasks are

$$m = 1.02,\ l = 1,\ l_p = 0.7\ (l_s = 0.3),\ g = 9.81$$



Task a): Simulate the motion of the pendulum with the following initial conditions and plot φ over t:

i) $\varphi_o = \frac{\pi}{6},\ \dot{\varphi}_o = 0,\ d = 0.2,\ \varphi_p = -\frac{\pi}{12},\ t\ \varepsilon\ [0,10]$

ii) $\varphi_o = -\frac{\pi}{6},\ \dot{\varphi}_o = 0,\ d = 0.1,\ \varphi_p = -\frac{\pi}{12},\ t\ \varepsilon\ [0,10]$
(the pin is left of the pendulum)

Task b): The equations can be linearized giving the linear model

$$m\,l\,\ddot{\varphi}_L = -m\,g\,\varphi_L - d\,l\,\dot{\varphi}_L$$

Implement the linear model and compare the results of non-linear and linear model by plotting φ and φ_L together and the deviation over t for

$$\varphi_o = \varphi_{Lo} = \frac{\pi}{12},\ \dot{\varphi}_o = \dot{\varphi}_{Lo} = 0,$$

$$\varphi_p = -\frac{\pi}{24},\ d = 0.2,\ t\ \varepsilon\ [0,10].$$

Indicate, whether the language permits comparison of sequential simulation runs of the different models, or whether the two models must be run simultaneously as a single simulation.

Task c): For

$$\varphi_o = \frac{\pi}{6},\ \varphi_p = -\frac{\pi}{12},\ d = 0.2$$

determine the initial angular velocity $\dot{\varphi}_o$ so that the maximum angle of the shortened pendulum φ reaches exactly $-\pi/2$. Indicate experimentation commands or model changes for automatic or manual variation of initial angular velocity $\dot{\varphi}_o$.

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