A fully Numerical Solution of ARGESIM Comparison "C5 - Two State Model" with SIMULINK

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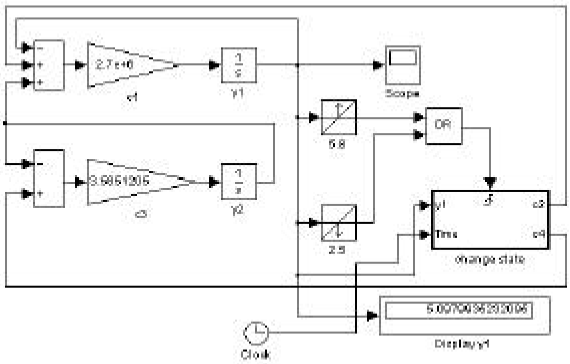
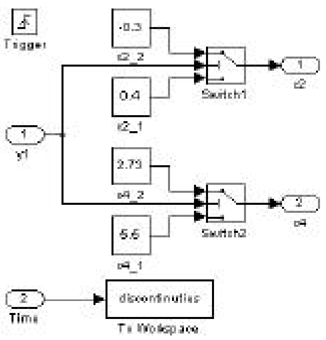
Simulator: SIMULINK is MATLAB's software package for graphical modeling, simulating and analyzing dynamic models. It provides a graphical user interface for building block diagrams.

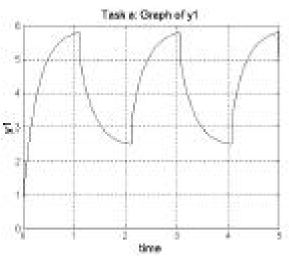
Model: The model uses SIMULINK's predefined standard blocks. For modeling the differential equations two `Integrators`, `Gains` and `Sums` are used. The detection of the change of states is implemented by two `Hit Crossing` blocks. If the value of y_1 rises above 5.8 or falls below 2.5 one of the blocks outputs 1, otherwise 0.



In the case of output 1, a triggered subsystem is executed. The subsystem changes the values of c_2 and c_4 by using `Switches` which pass through different values depending on y_1.



Task a, b: The following table shows the results for the located discontinuities and the final value of y_1 for different solvers (relative tolerance of 10^{-10}.



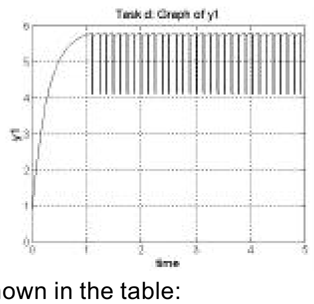
As the system is stiff, the `ode45` solver, which is based on an explicit Runge-Kutta formula, is inefficient and the last discontinuity is not found. The solvers, which are suitable for stiff systems (`ode15s`, `ode23s`, `ode23t`, `ode23tb`) are accurate enough to locate the last discontinuity. The table shows the results for `ode15s`, a variable order multistep solver based on numerical differentiation formulas and `ode23s`, a modified Rosenbrock method but the other two stiff solvers also provide similar results.

solver	ode45 Dormand-Prince	ode15s stiff/NDF	ode23s stiff/Mod. Rosenb.
t_0	0.00000021204414	0.00000021204049	0.00000021204189
t_1	1.10830375286938	1.10823990341505	1.10826751481646
t_2	2.12968573520500	2.12953629209048	2.12959805425123
t_3	3.05415418062369	3.05394045301652	3.05402788435275
t_4	4.07553425048787	4.07523684992900	4.07535852984632
t_5		4.99964071144095	4.99978874716609
$y_1(5)$	5.79999923704293	5.09658601719750	5.09799352320946

Task c: When using the relative error tolerance 10^{-14} SIMULINK displays a warning that the value is too small and automatically sets it to the value $2.842170943040401*10^{-14}$.

solver	ode23s (stiff/Mod. Rosenbrock)		
rel. tol.	1.E-06	1.E-10	1.E-14
t_0	0.00000021203842	0.00000021204189	0.00000021204189
t_1	1.10819703097179	1.10826751481646	1.10826702210142
t_2	2.12943121479381	2.12959805425123	2.12959815870731
t_3	3.05378375726469	3.05402788435275	3.05402798906175
t_4	4.07501989547822	4.07535852984632	4.07535910244564
t_5	4.99937437981031	4.99978874716609	4.99978931998371
$y_1(5)$	5.09405508729644	5.09799352320946	5.09799897169501

Task d: The changing of the state 2 parameter values and the switching condition results in high frequent oscillation behaviour of y_1. All used solvers compute 63 discontinuities. The first and last discontinuities and the final value of y_1 are shown in the table:



solver	ode45	ode15s	ode23s
t_0	0.00000000962055	0.00000000962066	0.00000000962046
t_1	1.10830389685258	1.10823958567604	1.10826680849999
t_2	1.12172730465457	1.12166325375780	1.12169046605979
...	...	...	...
t_{61}	4.92306167126274	4.92251064904719	4.92274184583829
t_{62}	4.93648552292577	4.93593431333147	4.93616550342221
$y_1(5)$	5.78039335464986	5.78063127592552	5.78053132948846

C5 Classification: Numerical Approach

Simulator: MATLAB / SIMULINK Rel. 13