C5 Two State Model – Maple 7

Semi-Analytical approach

Simulator: Maple 7 is a computer algebra system which is mainly used for symbolic calculation but it also includes numerical features. It provides a vast library of built-in functions and operations and allows arbitrary high accuracy by making it possible to change the numbers of digits carried in floats.

Model: This solution makes use of Maple's integrated ODE solver `dsolve`. The function `dsolve` is able to handle different types of problems by using classification and symmetry methods. In this special case it classifies the system as a system of first order linear differential equations and gives the solution in the analytical form $y_1(t)=k_1\cdot exp(-?_1\cdot t)+k_2\cdot exp(-?_2\cdot t)+c$. As Maple uses symbolical calculation, first the equations are solved with unknown parameters c_1, c_2, c_3 and c_4. The particular values are substituted into the solution. The search for the time instants of the change of states is realised by a modified bisection method. First the discontinuity is searched for with step size 0.01. Then the step size is divided by 10 and the last interval is inspected again. The process is iterated until a given step size (`step_bound`) is reached. This method makes it possible to find the discontinuities in reasonable time and with a sufficient accuracy.

```
> sys:={diff(y1(t),t)=c1*(y2(t)+c2-y1(t)),
diff(y2(t),t)=c3*(c4-y2(t)),
y1(t_init)=y1_init,y2(t_init)=y2_init};
> sol:=dsolve(sys,{y1(t),y2(t)});
> y1:=subs(sol,y1(t)); y2:=subs(sol,y2(t));
> c1:=2.7*10^6; c3:=3.5651205;
> c2_1:=0.4; c4_1:=5.5; c2_2:=-0.3; c4_2:=2.73;
> t_init:=0; y1_init:=4.2; y2_init:=0.3;
> z:=0; i:=0; state:=1; step_bound:=10^(-10);
> while (z<5) do i:=i+1;
y1||i:=subs(c2=c2_||state,c4=c4_||state,y1);
y2||i:=subs(c2=c2_||state,c4=c4_||state,y2);
step:=0.01;
if state = 1 then
 while step>=step_bound do
  while ((evalf(subs(t=z,y1||i))<5.8) and (z<5)) do
   z:=z+step;
  end do;
  z:=z-step; step:=step/10;
 end do; else ...
z:=z+step*10; disc||i:=z;
state:= (state mod 2) + 1; t_init:=z;
y1_init:=evalf(subs(t=z,y1||i));
y2_init:=evalf(subs(t=z,y2||i));
end do:
```

Results Task a: Simulation in time domain- switching times and final value. The table at the right shows the time for every located discontinuity and the value of $y_1(5)$.

t_1	1.10830616780000
t_2	2.12968535520000
t_3	3.05415290710000
t_4	4.07553209450000
t_5	4.99999964640000
$y_1(5)$	5.36944281876194

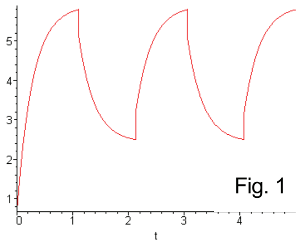

Fig. 1

Figure 1 shows the graph of y_1 over time. The result was calculated with `step_bound` of 10^{-10} and `Digits: =15`.

Task b: Influence of accuracy on solution. Accuracy can be varied in this model by two parameters. First by Maple's environment variable `Digits`, which controls the number of digits that Maple uses when calculating with floating-point numbers and second by the variable `step_bound`, which determines the minimum step size for the time loop. The next tables show the results for different values of these parameters.

$y_1(5)$:

step	Digits=15	Digits=20	Digits=30
10^{-6}	5.79999908 287094	5.79999908287 09388255	5.79999908287093 882550066486064
10^{-10}	5.36944281 876194	5.36944281876 19019434	5.36944281876190 194324447677611
10^{-14}	5.36931208 829560	5.36931214700 63303554	5.36931214700633 035514320327283

Time of last discontinuity:

step	Digits=15	Digits=20	Digits=30
10^{-6}	-	-	-
10^{-10}	4.9999996464	4.9999996464	4.9999996464
10^{-14}	4.9999996462 2025	4.9999996462 2034	4.9999996462 2034

With a step size of 10^{-6} the last discontinuity is not found. Step size 10^{-10} is already small enough to produce result with adequate accuracy.

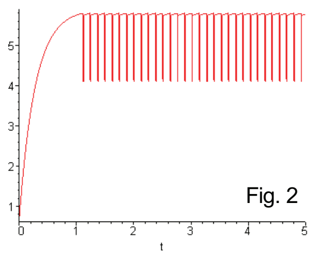

Fig. 2

Task d: Highly oscillating solution. The change of the state 2 parameter values causes a high frequent oscillating behaviour of y_1 with 62 discontinuities. The table below shows the first and last discontinuities computed with `step_bound:=10^-14` and `Digits:=20`. The final value of y_1 is 5.7804025205614051442.

t_1	1.10830616777114	t_2	1.12172996789144
t_3	1.23546396574812	t_4	1.24888776586842
...	...	...	...
t_{59}	4.79588230910372	t_{60}	4.80930610922402
t_{61}	4.92304010708071	t_{62}	4.93646390720101

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