C5 Two State Model – Maple 7

Fully Analytical approach

Simulator: Maple 7 is a computer algebra system which is mainly used for symbolic calculation but it also includes numerical features. It provides a vast library of built-in functions and operations and allows arbitrary high accuracy by making it possible to change the numbers of digits carried in floats.

Model: This solution doesn't use Maple's integrated ODE solver `dsolve` (although possible): it shows how one can implement an algorithm for solving a system of differential equations. First the eigenvalues and the eigenvectors of the linear homogeneous system of equations are determined, to create a solution basis for the homogeneous system. Therefore Maple's function `Eigenvals` is used.

```
> A:=matrix([[-c1,c1],[0,-c3]]);
> b_1:=matrix(2,1,[c1*c2_1,c3*c4_1]);
> b_2:=matrix(2,1,[c1*c2_2,c3*c4_2]);
> lambda:=evalf(Eigenvals(A,vecs));
> J:=array([[exp(lambda[1]*t),0],
          [0,exp(lambda[2]*t)]]);
> L:=multiply(vecs,J);
```

Next the inhomogeneous part is solved by a simple system of equations because the inhomogenity is constant. (The Matrix doesn't have to be inverted).

```
> const1:=solve({A[1,1]*k1_1+A[1,2]*k2_1=b_1[1,1],
    A[2,1]*k1_1+A[2,2]*k2_1=b_1[2,1]},{k1_1,k2_1});
> const2:=solve({A[1,1]*k1_2+A[1,2]*k2_2=b_2[1,1],
    A[2,1]*k1_2+A[2,2]*k2_2=b_2[2,1]},{k1_2,k2_2});
> Eq:=geneqns(L,[v1,v2],
            vector([y1_init+k1_1,y2_init+k2_1]));
> Eqt:=subs(t=t_init,Eq);
> lsg1:=solve(Eqt,{v1,v2});
> Eq:=geneqns(L,[v1,v2],
            vector([y1_init+k1_2,y2_init+k2_2]));
> Eqt:=subs(t=t_init,Eq);
> lsg2:=solve(Eqt,{v1,v2});
> t_init:=0; y1_init:=4.2; y2_init:=0.3;
> z:=0; i:=0; state:=1;
```

The time instants of the change of states are determined by a modified bisection method, whereby the smallest step size can be constituted.

```
> while (z<5) do i:=i+1;
  sol:=multiply(L,matrix(2,1,[v1,v2])):
 funcy1||i:=unapply(sol[1,1]-k1_||state,t):
 funcy2||i:=unapply(sol[2,1]-k2_||state,t):
 step:=0.01;
 if state = 1 then
   while step>=step_bound do
     while ((funcy1||i(z)<5.8) and (z<5)) do
       z:=z+step;
     end do;
     z:=z-step;
     step:=step/10;
   end do;  else ...
 z:=z+step*10; disc||i:=z;
 state:=(state mod 2)+1; t_init:=z;
 y1_init:=funcy1||i(z); y2_init:=funcy2||i(z);
end do:
```

Results Task a: Simulation in time domain- switching times and final value. The table at the right shows the time points for all detected discontinuities and also the value of $y_1(5)$. The associated graph of y_1 is shown in Fig. 1. The result was calculated with `Digits: =15` and a `step_bound` of 10^{-10}.

t_1	1.10830616780000
t_2	2.12968535520000
t_3	3.05415290710000
t_4	4.07553209450000
t_5	4.99999964640000
$y_1(5)$	5.36944281876193

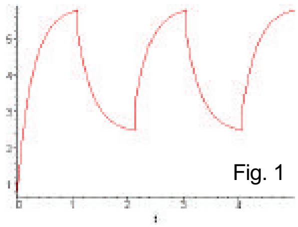

Fig. 1

Task b: Influence of accuracy on solution. The model accuracy can be varied by two parameters. First by changing the number of digits of the used floating-point numbers (Maple's environment variable `Digits`.) Second by the variable `step_bound`, which determines the minimum step size for the time loop. In the next table the results for different values of these parameters are shown. Because there are nearly no differences between values by changing `Digits` (Digits = 30 used), only the `step_bound` is varied.

step	$y_1(5)$	Time of last discontinuity
10^{-6}	5.79999908287093882 550066486064	4.075535
10^{-10}	5.36944281876190194 324447677610	4.9999996464
10^{-14}	5.36931214700633035 514320327281	4.99999964622034

With a step size of 10^{-6} one discontinuity less is found.

Task d: Highly oscillating solution. Figure 2 shows the graph of y_1 for the system with changed state 2 parameter values. It's high frequently oscillating and forces 62 discontinuities. The first and last discontinuities (see table at right) are computed with following accuracy parameters: `step_bound:=10`$^{-14}$ `Digits:=20`

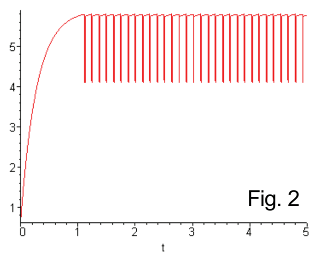

Fig. 2

t1	1.10830616777114
t2	1.12172996789144
...	...
t61	4.92304010708071
t62	4.93646390720101

The final value of y_1 is 5.7804025205614051443.

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