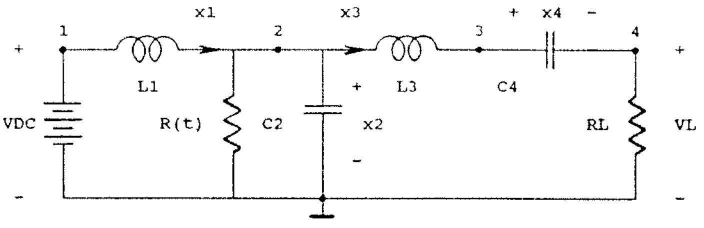
Comparison 3: Analysis of a generalized class-E amplifier

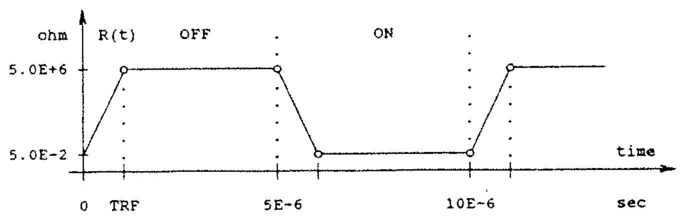
This example is taken from the electrical engineering world.

The basic class-E power amplifier was introduced by N.O. Sokal and A.D. Sokal in their classic paper from 1975 [1]. It is a switching-mode amplifier that operates with zero voltage and zero slope across the switch at switch turn-off. The actual numerical example is taken from J.C. Mandojana, K.J. Herman and R.E. Zulinski [2]. They use the following equivalent circuit of a generalized class-E amplifier as a test example for a procedure to evaluate steady state boundary conditions by means of MATLAB:



The component values are:
VDC = 5 volt, L1 = 79.9E-6 henry, C2 = 17.9E-9 farad, L3 = 232.0E-6 henry, C4 = 9.66E-9 farad and RL = 52.4 ohm.

The time dependent resistor R(t) models the active device acting as a switch with an ON-resistance of 0.05 ohm and an OFF-resistance of 5.0E + 6 ohm. An extreme ON-resistance of value zero ohm will of course result in a pathological system i.e. the old story of what happens when an ideal capacitor with a certain charge is suddenly short circuited. Furthermore the DC voltage source will be short circuited through the ideal coil L1. As a function of time R(t) is given in the following graph:



The duty ratio is 50%. The period is 10E-6 seconds (frequency 100 kHz). The rise/fall time is TRF = 1E-15 seconds.

The equations describing the circuit may be the state-equations where inductor currents and capacitor voltages are chosen as system variables. By using the Kirchhoff voltage and current laws we get the following differential equations:

$$\begin{aligned}
L1*dx1/dt &= -x2 + VDC\\
C2*dx2/dt &= +x1 - x2/R(t) - x3\\
L3*dx3/dt &= +x2 - RL*x3 - x4\\
C4*dx4/dt &= +x3
\end{aligned}$$

where the variables are as follows: x1 = IL1 (the current of L1), x2 = VC2 (the voltage of C2), x3 = IL3 (the current of L3) and x4 = VC4 (the voltage of C4). Note that normally the setup of state equations demands a topological analysis of the circuit excluding some inductor currents and capacitor voltages as candidates for system variables (e.g if there is a loop of N capacitors then only N-1 of these may be given an arbitrary initial charge).

The following tasks should be performed:

(a) Calculation of the eigenvalues of the system in the ON-period: R(t) = 0.05 ohm and in the OFF-period: R(t) = 5E + 6 ohm.

(b) Simulation of the system over the time interval [0, 100E-6] sec with the zero-solution as initial state. Time curves of the state variables, the current in the switch resistor IR(t) = x2/R(t) and the output voltage VL = x3*RL are wanted.

(c) A parameter variation study over the time interval [0, 9E-6] sec with initial solution equal to the final solution at 100E-6 sec from task (b). The rise/fall time TRF should be varied through the values: 1E-15, 1E-11, 1E-9, 1E-7 sec. The phase plane curves of dx3/dt = VL3 as a function of x3 = IL3 i.e the voltage difference V2-V3 as a function of the current IL3 are wanted. Time curves of the current in the switch resistor IR(t) = x2/R(t) and the output voltage VL = x3*RL are wanted.

References

[1] Nathan O. Sokal and Alan D. Sokal, Class E - A New Class of High-Efficiency Tuned Single-Ended Switching Power Amplifiers, IEEE Journal of Solid-State Circuits, Vol. SC-10, No. 3, June 1975, pp. 168-176.

[2] Julio C. Mandojana, Kelly J. Herman and Robert E. Zulinski, A Discrete/Continuous Time-Domain Analysis of a Generalized Class E Amplifier, IEEE Transactions on Circuits and Systems, Vol. 37, No. 8, August 1990, pp. 1057-1060

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