EXCEL as Simulation Tool for ARGESIM Benchmark C21 'State Events and Structural Dynamic Systems', Case Study RLC with Diode

Clemens Karner, Michael Farthofer, Ludwig Fellinger, Gebhard Fellinger, Felix Breitenecker*

[1]Institute of Analysis and Scientific Computing, TU Wien, Wiedner Hauptstrasse 8-10, 1040 Vienna, Austria
*felix.breitenecker@tuwien.ac.at

SNE 35(3), 2025, 159-170, DOI: 10.11128/sne.35.ebn21.10746
Received: 2023-11-23; Improved: 2024-08-15
Revised: 2025-06-01; Accepted: 2025-07-01
SNE - Simulation Notes Europe, ARGESIM Publisher Vienna
ISSN Print 2305-9974, Online 2306-0271, www.sne-journal.org

Abstract. Very often EXCEL is called the most-used simulator – but also for continuous ODE systems? Generally, EXCEL allows easy implementation of recursive formula, as for the EULER solver (there exist also ODE solver macros for EXCEL). This student project reports on experiences with a basic recursive EULER solver for ARGESIM Benchmark C21, case study 'RLC with Diode, and documents also the model set up for educational purposes.

For the event handling of diode switching, two previous steps of the solver had to be observed – with reasonable results. Some problems occurred with the numerical treatment of the formula, because of the big range of parameters, leading to numerical extinction errors, especially in case of the Shockley formula interpolation.

For the reasonable results, a very fine grid had to be used – an EXCEL table with 40.000 rows (and some for initialisation. The answer: EXCEL can also be used for continuous systems with events, but cannot be really recommended.

Introduction – Basic Modelling

ARGESIM Benchmark C21 ''State Events and Structural Dynamic Systems', Case Study RLC with Diode is based on the dynamics of a simple RLC circuit with a diode in parallel (Figure 1).

The RLC circuit is made up from idealized parts, with exception of the diode: an alternating voltage source U_0, two resistors R_1, R_1, an inductor L and a capacitor C, all with continuous description between voltage and current.

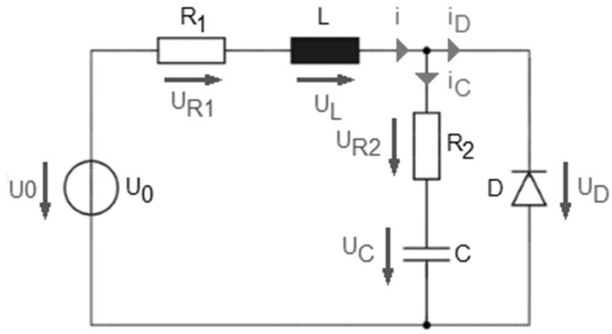

Figure 1: RLC Circuit with diode in parallel, counting direction for currents and voltages.

The basic dynamics are given by the active elements, by inductor L and a capacitor C:

$$\frac{di_L}{dt} = \frac{1}{L}u_L \qquad \frac{du_C}{dt} = \frac{1}{C}i_C$$

So generally, two coupled differential equations have to be solved. A basic solution approach is the Euler ODE solver with an appropriate stepsize $\Delta t = t_{n+1} - t_n$:

$$i_{L,n+1} = i_{L,n} + \Delta t \cdot \frac{1}{L}u_{L,n} = \Delta t \cdot f_n(i_{L,n}, u_{C,n})$$

$$u_{C,n+1} = u_{C,n} + \Delta t \cdot \frac{1}{C}i_{c,n} = \Delta t \cdot g_n(i_{L,n}, u_{C,n})$$

EXCEL allows to implement these recursive formulas easily step by step in consecutive rows (Table 1).

0	t_0	$i_{L,0}$	$u_{C,0}$
…	…	…	…
n-1	t_{n-1}	$i_{L,n-1} = \cdots$	$u_{C,n-1} = \cdots$
n	t_n	$i_{L,n} = i_{L,n-1} +$ $+\Delta t \cdot f_{n-1}$	$u_{C,n} = u_{C,n-1} +$ $+\Delta t \cdot g_{n-1}$
n+1	t_{n+1}	$i_{L,n+1} = i_{L,n} +$ $+\Delta t \cdot f_n(i_{L,n}, u_{C,n})$	$u_{C,n+1} = u_{C,n} +$ $+\Delta t \cdot g_n(i_{L,n}, u_{C,n})$
…	…	…	…

Table 1: Euler solver steps in EXCEL.

For the diode, the benchmark suggests three different model approaches: an ideal diode, a diode with Shockley dynamics, and a diode with interpolated/approximates Shockley dynamics. Generally, the diode action – conducting or locking changes the 'update'-equations f_n and g_n, requiring IF – THEN - ELSE constructs in the EXCEL implementation.

1 RLC Model – Ideal Diode

The characteristic behaviour of an ideal diode can be described by a discontinuous function consisting of two parts. One part is for the locking phase; the other is for the conducting phase of the diode.

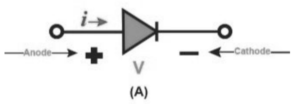

Figure 2: Electrical symbol of a diode (detailed)

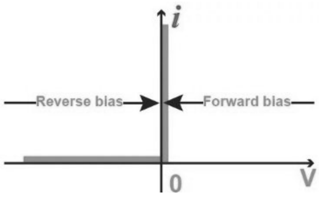

Figure 3: Characteristics of an ideal diode.

The mathematical description for the ideal diode is given by switching between locking phase $u_D > 0$ and conduction phase $u_D \leq 0$:

$$i(u_D) = \begin{cases} 0, \; u_D > 0 \\ i(t), u_D \leq 0 \end{cases}$$

1.1 RLC Circuit - ideal diode in locking phase

In locking phase, the ideal diode is reverse biased and therefore no current i_D can pass. Figure 4 shows the circuit with directions for current (green circles).

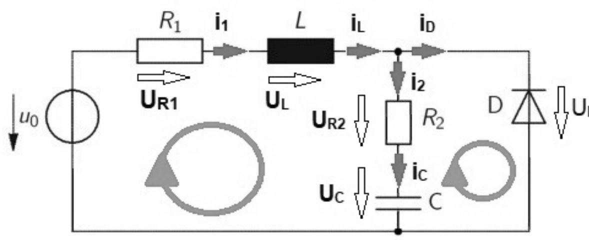

Figure 4: RLC circuit in locking phase.

Applying Kirchhoff's mesh rule (green circles) gives:

$$0 = -U_0 + u_{R1} + u_L + u_{R2} + u_C$$

$$0 = u_D - u_{R2} - u_C$$

Kirchhoff's junction rule in case of $i_D = 0$ ends up in the following identities:

$$i_1 = i_L = i_2 = i_C$$

Using now the dynamic equations for inductor L and capacitor C, and Ohm's law for the resistances R_1 and $R2$

$$\frac{di_L}{dt} = \frac{1}{L} u_L \qquad \frac{du_C}{dt} = \frac{1}{C} i_C$$

$$U_{R1} = i_1 \cdot R_1, \qquad U_{R2} = i_2 \cdot R_2$$

allows to rearrange the equations for the voltages

$$u_L = U_0 - u_{R1} - u_{R2} - u_C$$

$$u_L = U_0 - i_1 \cdot (R_1 - R_2) - u_C$$

and with substitution of U_L by u_1 to derive the full dynamic description of the system:

$$\frac{di_1}{dt} = \frac{1}{L} \cdot (U_0 - i_1 \cdot (R_1 - R_2) - u_C)$$

$$\frac{du_C}{dt} = \frac{1}{C} i_1$$

$$u_D = i_1 \cdot R_2 + u_C, \; u_D > 0$$

It is to be noted, that the diode voltage u_D does not appear in the state equations for i_1 and u_c, but the equation for u_D controls the locking phase $u_D > 0$

The Euler ODE solver can be applied now to the state equations for i_1 and u_c at an equidistant grid

$$t_0 < t_1 < t_2 \dots < t_{n-1} < t_n < t_{n+1} \dots < t_{end-1} < t_{end}$$

with stepsize $\Delta t = t_{n+1} - t_n$:

$$i_{1_{n+1}} = i_{1_n} + \Delta t \cdot \left(\frac{1}{L} \cdot (U_{0_n} - i_{1_n} \cdot (R_1 - R_2) - u_{C_n}) \right)$$

$$u_{C_{n+1}} = u_{C_n} + \Delta t \cdot \left(\frac{1}{C} \cdot i_{1_n} \right)$$

$$u_{D_n} = i_{1_n} \cdot R_2 + u_{C_n}, \; u_D > 0$$

$$u_{D_{n+1}} = i_{1_{n+1}} \cdot R_2 + u_{C_{n+1}}, \; u_D > 0$$

This set of recursive equations is suitable for implementation in EXCEL. Additionally to the state update from t_n to t_{n+1}, the relation for u_{D_n} or $u_{D_{n+1}}$ resp., must be checked for positivity.

In case of negativity of $u_{D_{n+1}}$ to next step takes place in the conduction phase. The exact time instant t_{change} of phase change lies between t_n and t_{n+1} and is roughly approximated by the next update step.

1.2 Circuit - ideal diode in conducting phase

In conducting phase, the ideal diode is forward biased and therefore current i_D can pass. Due to this fact the ideal diode behaves like an ideal conductor in conducting phase and the voltage u_D will be zero.

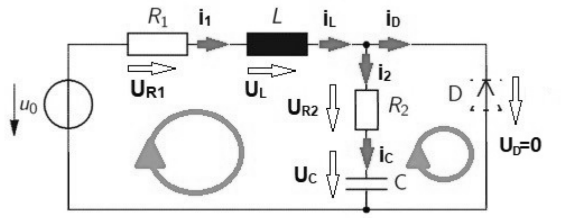

Figure 5: RLC circuit in conducting phase.

Applying Kirchhoff's mesh rule (green circles) again gives:

$$0 = -U_0 + u_{R1} + u_L + u_{R2} + u_C$$

$$0 = u_D - u_{R2} - u_C$$

But Kirchhoff's junction rule in conducting phase with $i_D \neq 0$ now gives:

$$0 = i_L - i_2 - i_D \quad 0 = i_1 - i_L \quad 0 = i_2 - i_C$$

Using now as before the dynamic equations for inductor L and capacitor C, and Ohm's law for the resistances R_1 and $R2$ allows to rearrange the equations for the voltages with different result:

$$u_L = U_0 - u_{R1} - u_{R2} - u_C$$

$$u_L = U_0 - i_1 \cdot R_1 - u_D$$

In the conducting phase the ideal diode is a perfect conductor and no voltage drop will appear

$$u_L = U_0 - i_1 \cdot R_1$$

so that with substitution of i_L by i_1 the full dynamic description in conducting phase becomes

$$\frac{di_1}{dt} = \frac{1}{L} \cdot (U_0 - i_1 \cdot R_1)$$

$$\frac{du_C}{dt} = -\frac{1}{C \cdot R_2} \cdot u_C$$

$$i_D = i_1 + \frac{1}{R_2} \cdot u_C$$

Nevertheless, U_D is zero in conduction phase we have to recognize phase changes by calculating U_D also in this phase:

$$u_D = (i_1 - i_D) \cdot R_2 + u_C$$

As before, the Euler ODE solver can be applied now to the state equations for i_1 and u_c at an equidistant grid

$$t_0 < t_1 < t_2 \ldots < t_{n-1} < t_n < t_{n+1} \ldots < t_{end-1} < t_{end}$$

with stepsize $\Delta t = t_{n+1} - t_n$:

$$i_{1_{n+1}} = i_{1_n} + \Delta t \cdot \left(\frac{1}{L} \cdot \left(U_{0_n} - i_{1_n} \cdot R_2\right)\right)$$

$$\mathrm{F}u_{C_{n+1}} = u_{C_n} + \Delta t \cdot \left(-\frac{1}{C \cdot R_2} \cdot u_{C_n}\right)$$

$$i_{D_n} = i_{1_n} + \frac{1}{R_2} \cdot u_{C_n}, i_{D_{n+1}} = i_{1_{n+1}} + \frac{1}{R_2} \cdot u_{C_{n+1}}$$

$$u_{D_n} = \left(i_{1_n} - i_{D_n}\right) \cdot R_2 + u_{C_n}$$

$$u_{D_{n+1}} = \left(i_{1_{n+1}} - i_{D_{n+1}}\right) \cdot R_2 + u_{C_{n+1}}$$

If the diode voltage becomes nonzero, $u_{D_{n+1}} \neq 0$, the next step takes place in the locking phase. The exact time instant t_{change} of phase change lies between t_n and t_{n+1} and is roughly approximated by the next update step.

1.3 Phase change Locking - Conducting

The diode voltage u_D is the decisive factor which implies whether the diode is in locking phase or conducting phase. The sign change or the change to zero of u_D are so-called state events, unknown in time, which decide that in each phase (locking or conducting) a different set of equations are used for the calculation of the voltage and current in the circuit.

To recognize the state event *start of the conducting phase* (SCP), the diode voltage u_D in the locking phase must be observed. When the diode voltage u_D in the locking phase sinks below zero and changes its sign, the state event *start of conducting phase* is triggered. From now on the diode is in conducting phase and different equations including the equation for calculating u_D itself must be used. This can be expressed by a so-called state event determination equation

$$u_D \overset{!\,-}{=} i_1 \cdot R_2 + u_C \qquad \text{SCP}$$

determining the time instant t_{SCP}, where SCP takes place, i.e. where the equation has a zero (from plus to minus):

$$U_D(t_{SCP}) = i_1(t_{SCP}) \cdot R_2 + u_C(t_{SCP}) = 0$$

Obviously, more SCP events occur, with event times $t_{SCP,k}$, alternatingly with SLP events at $t_{SLP,j}$.

And similarly, to recognize the state event *start of the locking phase* (SLP) the diode voltage u_D in the conducting phase must be observed. When the diode voltage u_D of the conducting phase changes its sign from negative to positive the state event *start of the locking phase* is triggered. From now on the set of equations for the locking phase must be used.

This can also be expressed by a state event determination equation

$$u_D \overset{!+}{=} (i_1 - i_D) \cdot R_2 + u_C \qquad \text{SLP}$$

determining the time instant t_{SLP}, where SCP takes place, i.e. where the equation has a zero (from minus to plus):

$$u_D(t_{SCP}) = \left(i_1(t_{SlP}) - i_D(t_{SlP})\right) \cdot R_2 + u_C(t_{SlP}) = 0$$

And again obviously, more SLP events occur, with event times $t_{SLP,j}$, alternatingly with SCP events at $t_{SCP,k}$.

During time solution in the locking phase and in the conducting phase, the state events SCP and SLP must be found. They take place between neighbouring steps of the ODE solver,

$$t_i \le t_{SLP} \le t_{i+1}, \text{ or } t_m \le t_{SCP} \le t_{m+1} \text{ resp.}$$

Some simulation systems provide algorithms for finding these event times iteratively.

In EXCEL, we have fixed time steps, so one can simply approximate the event time with the next step time, where it is detected, i.e. the event SCP or SLP takes place between t_i and t_{i+1}, the event time is fixed with

$$t_{SLP} = t_{i+1} \text{ . or } t_{SCP} = t_{m+1}.$$

But unfortunately, using the sign change of the diode voltage u_D as an indicator of phase change works only in theory also here. In this case study, the sign change is more complex. Due to the fact that in conducting phase the ideal diode behaves like a perfect conductor the diode voltage u_D equals zero in conducting phase. So, the above mechanism for phase change can only be used from locking phase to conducting phase.

From conducting phase to locking phase, the classical mechanism fails. it's not possible to apply these definitions, because there is only a transition from u_D is zero to u_D is positive – one has to observe in this case not only one time step t_i and t_{i+1}, but two time steps t_{i-1}, t_i and t_{i+1}, to determine SLP with $t_{SLP} = t_{i+1}$ properly.

1.4 EXCEL implementation – ideal diode model

In the following, the overall structure of the developed spreadsheet and the ODE solve update with event detection is described.

At the top of the spreadsheet circuit parameters (electrical characteristics), initial conditions and simulation parameters (e.g. time step) are initialized (Table 2).

The benchmark definition and the characteristics of the input voltage U_0 (high frequency excitation) suggest a simulation horizon of $5 \cdot 10^{-4}$ sec, so we decided for a time step $\Delta t = 1.25 \cdot 10^{-8}$ sec, resulting in indeed 40000 EXCEL rows. This fine resolution would not be necessary for the Euler solver, but for approximating the phase change time instants it was appropriate. A finer resolution would make problems with numerics (extinction error).

	A	B	C	D	E	
1						
2						
3	Parameter			Simulationsparameter		
4	R1 [ohm]	1000		Zeilenzahl [1]	40000	
5	R2 [ohm]	20		Simulationsdauer [s]	0,0005	
6	L [H]	2,52E-05		dt [s]	1,25E-08	
7	C [F]	1,00E-07		t0 [s]	0	
8	Is [A]	1,00E-08				
9	Ut [V]	0,026				
10	f [Hz]	142857				
11						
12						
13						
14	Initial Conditions					
15	i1_0 [A]	0				
16	Uc_0 [V]	0				
17						
18						
19						
20						
21						
22						
23	Start with Locking Phase					
24						
25	t	U0	i1	Uc	Ud	Id
26	0	0	0	0	0,00E+00	
27	1,25E-08	0,01121973	0,00E+00	0,00E+00	0,00E+00	
28	0,000000025	0,02243804	5,57E-06	0,00E+00	1,11E-04	
29	3,75E-08	0,03365353	1,39E-05	6,96E-07	2,78E-04	
30	0,00000005	0,04486479	2,36E-05	2,43E-06	4,73E-04	
31	6,25E-08	0,05607039	3,39E-05	5,37E-06	6,83E-04	

Table 2: Overall structure of EXCEL implementation for ideal diode circuit.

Below the parameter definitions of the spreadsheet, the columns for time t , the excitation voltage U_0 , the current through inductor i_1, the voltage in capacitor u_C, the voltage drop in diode u_D and the current through diode i_D start (Table 3). The EXCEL setup of the recursive equations for the EULER solver follows the principle given in Table 1.

	A	B	C	D	E	F
22						
23	Start with Locking Phase					
24						
25	t	U0	i1	Uc	Ud	Id
26	0	0	0	0	0,00E+00	0
27	1,25E-08	0,0112197	0,00E+00	0,00E+00	0,00E+00	0
28	0,000000025	0,022438	5,57E-06	0,00E+00	1,11E-04	0
29	3,75E-08	0,0336535	1,39E-05	6,96E-07	2,78E-04	0
30	0,00000005	0,0448648	2,36E-05	2,43E-06	4,73E-04	0
31	6,25E-08	0,0560704	3,39E-05	5,37E-06	6,83E-04	0
32	0,000000075	0,0672689	4,46E-05	9,61E-06	9,01E-04	0
33	8,75E-08	0,078459	5,54E-05	1,52E-05	1,12E-03	0
34	0,0000001	0,0896392	6,63E-05	2,21E-05	1,35E-03	0
35	1,125E-07	0,1008081	7,72E-05	3,04E-05	1,57E-03	0
36	0,000000125	0,1119644	8,81E-05	4,00E-05	1,80E-03	0
37	1,375E-07	0,1231065	9,91E-05	5,10E-05	2,03E-03	0
38	0,00000015	0,1342331	1,10E-04	6,34E-05	2,26E-03	0

Table 3: Start of spreadsheet simulation.

In the first ten rows of the EXCEL simulation we assumed to be in locking phase and used the equations of the locking phase. No case distinctions between the phases have been made in these rows. It can be compared with a booting up of the simulation. This approach is necessary because of the recursive case distinction which needs the data of the rows before.

So e.g. the state update in row 30 (i.e. at t_5) for $i_{1,5}$ and $u_{C,5}$ are in pseudo-EXCEL notation:

```
C30=C29+dt*(1/L*(B29-C29*(R1+R2)-D29))
D30=D29+dt*(1/C*C29)
```

After this 'booting' for the solver update in the cells for i_1, u_C, u_D and i_D – i.e. in columns C, D, E and F – it is necessary to make use of if-then-else formula for calculating the values in a cell.

To determine the values in these columns, the information from three timesteps is needed for update to t_k, from t_{k-2} and t_{k-1} for current and voltage and from t_k for time and voltage input.

In cell C37, the current through inductor at timestep t_k is calculated. To update the current i_1 either the recursive formula in locking phase or the recursive formula in conducting phase must be applied. For the distinction between these two formulas the occurrence of phase change must be checked.

This is done by observing the diode voltage U_D of two timesteps before (t_{k-2} and t_{k-1}). If both are greater than zero (AND condition $u_D(t_{k-2}) > 0$ & $u_D(t_{k-2})$), the value for i_1 is calculated with equation of the locking phase. Otherwise, the equation of the conducting phase is used (Table 4, upper part).

	A	B	C	D	E	F
25	t	U0	i1	Uc	Ud	Id
35	1,125E-07	0,1008081	7,72E-05	3,04E-05	1,57E-03	0
36	0,000000125	0,1119644	8,81E-05	4,00E-05	1,80E-03	0
37	1,375E-07	0,1231065	`=WENN(UND(E35>0;E36>0);C36+1/$B$6*(B36-C36*($B$4+$B$5)-D36)*$E$6;C36+1/$B$6*(B36-C36*$B$4)*$E$6)`			
38	0,00000015	0,1342331	1,10E-04	6,34E-05	2,26E-03	0
39	1,625E-07	0,1453429	1,21E-04	7,72E-05	2,49E-03	0

	A	B	C	D	E	F
25	t	U0	i1	Uc	Ud	Id
34	0,0000001	0,0896392	6,63E-05	2,21E-05	1,35E-03	0
35	1,125E-07	0,1008081	7,72E-05	3,04E-05	1,57E-03	0
36	0,000000125	0,1119644	8,81E-05	4,00E-05	1,80E-03	0
37	1,375E-07	0,1231065	9,91E-05	`=WENN(UND(E35>0;E36>0);D36+1/$B$7*C36*$E$6;D36+1/($B$7*$B$5)*(-D36)*$E$6)`		
38	0,00000015	0,1342331	1,10E-04	WENN(Wahrheitstest; [Wert_wenn_wahr]; [Wert_wenn_falsch])		0
39	1,625E-07	0,1453429	1,21E-04	7,72E-05	2,49E-03	0

Table 4: Conditional computation of current and voltage depending on phase, ideal diode model.

The capacitor voltage u_C is calculated in cell D37. Similar to calculation of i_1 also here a case distinction between locking and conducting phase must be made. The case distinction is also based recursively on the values of $u_D(t_{k-2})$ and $u_D(t_{k-2})$, see Table 4, lower part.

In the general consideration wrt. phase change before, only one previous step was necessary. But here two steps must be used, because of determining the direction of change and taking into account that the diode voltage does not cross zero, it is positive or zero.

1.5 EXCEL simulation – results ideal diode model

The results for capacitor voltage u_C , diode voltage u_D and diode current i_D show the expected behaviour (Figure 6). It is obvious, that the ‘ideal’ diode is a simplified idealized diode. While the time course for capacitor voltage u_C seems reasonable, the time course for diode voltage u_D shows a jump (not possible reality), and the diode current i_D passes through in locking phase with a ‘spike’.

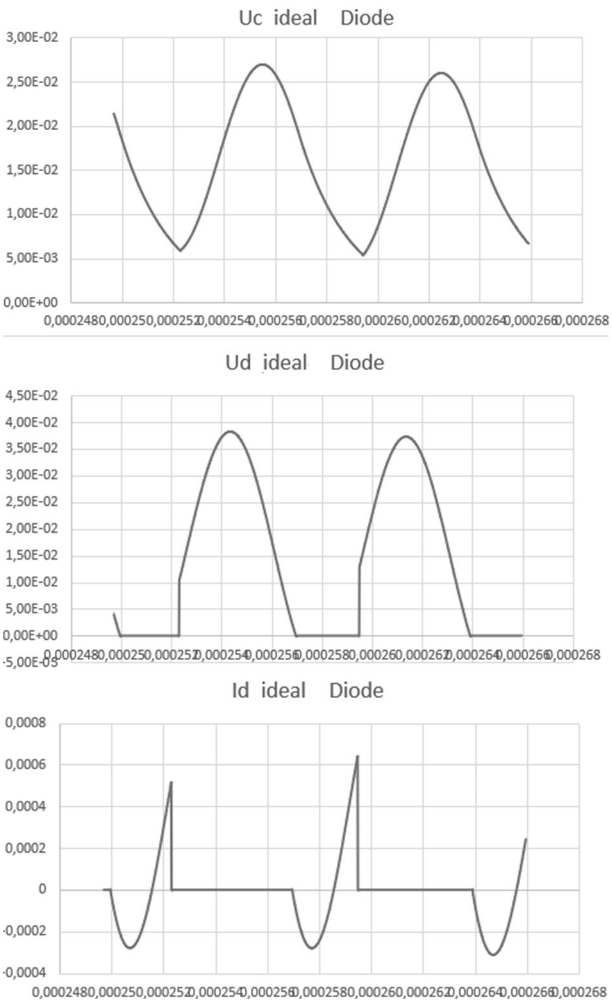

Figure 6: EXCEL simulation results for ideal diode model.

2 RLC Model – Interpolated Shockley Diode

A Shockley diode acts similarly to an ideal diode, but smoothens the phase change, and is much nearer to reality.

Basically, it is perfect insulator. When the applied voltage changes its sign and the Shockley diode is forward biased the behaviour is different to an ideal diode.

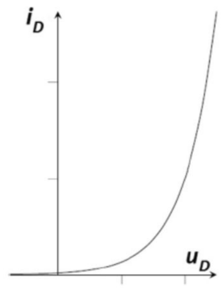

Figure 7: Shockley diode.

There is an exponential link between diode current and applied voltage (Figure 7).

The relation between current i_D and voltage u_D in a Shockley diode follows now a continuous function:

$$i(u_D) = \begin{cases} I_S * (e^{\frac{u_D}{U_T}} - 1), u_D \geq 0 \\ 0, u_D < 0 \end{cases}$$

In the conducting phase, there is an exponential-shaped (nonlinear) relation between diode voltage and diode current, which cannot be resolved directly. Generally, this relation adds an algebraic equation to the differential equations for capacity voltage and inductivity current, so that the system becomes a differential-algebraic equations – to be discussed in the next chapter.

2.1 Model with interpolated Shockley diode

The suggestion in this benchmark is, to approximate the nonlinear characteristics of the Shockley diode model by a table function, in this solution by three breakpoints, i.e.by two linear functions.

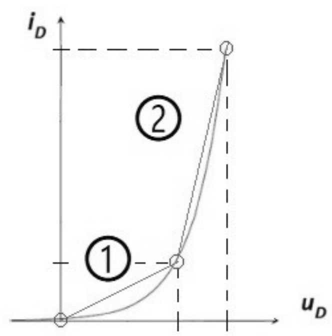

Figure 8: Shockley diode characteristics with approximation by linear functions.

In this approach the nonlinear current - voltage relation of the Shockley diode is approximated with linear functions. The diode voltage of the forward biased diode is split up in two parts. The first part ① extends between zero and 0,33 [V] and the second part ② extends between 0,33 [V] and 1 [V] (see Figure 8):

$$u_D = k_{low} \cdot I_d \qquad 0 \leq u_D \leq 0{,}33$$

$$u_D = k_{high} \cdot I_d + d \qquad 0{,}33 < u_D \leq 1$$

The model description with interpolated Shockley diode for the locking phase is the same as the model description with the ideal diode.

For the conducting phase with the interpolated Shockley diode, now the equations change. Following again Figure 5 (RLC circuit with conducting phase of diode) we end up with the following differential equations for capacitor voltage u_C and inductor current i_1:

$$\frac{di_1}{dt} = \frac{1}{L} \cdot (U_0 - i_1 \cdot (R_1 + R_2) + i_D \cdot R_2 - u_C)$$

$$\frac{du_C}{dt} = \frac{1}{C} \cdot (i_1 - i_D)$$

In these equations now the diode current appears, which is to be expressed and substituted by other voltages or currents resp.

Kirchhoff's mesh rule or junction rule, resp. gives the relations

$$0 = -u_C - i_2 \cdot R_2 + u_D$$

$$0 = -u_C - (i_1 - i_D) \cdot R_2 + u_D$$

Now we can use the linear relations of the interpolated Shockley characteristics $u_D = k_{low} \cdot I_d$ (region ①) or $u_D = k_{high} \cdot I_d + d$ (region ②):

$$i_D = \frac{(u_C + i_1 \cdot R_2)}{(R_2 + k_{low})} \qquad 0 \leq u_D \leq 0{,}33$$

$$i_D = \frac{(u_C + i_1 \cdot R_2 - d)}{(R_2 + k_{high})} \qquad 0{,}33 < u_D \leq 1$$

Inserting now the linear equations for the diode current i_D into the ODEs result for case ① in

$$\frac{di_1}{dt} = \frac{1}{L} \cdot (U_0 - i_1 \cdot (R_1 + R_2) + \frac{(u_C + i_1 \cdot R_2)}{(R_2 + k_{low})} \cdot R_2 - u_C)$$

$$\frac{du_C}{dt} = \frac{1}{C} \cdot (i_1 - \frac{(u_C + i_1 \cdot R_2)}{(R_2 + k_{low})})$$

and for case ② in:

$$\frac{di_1}{dt} = \frac{1}{L} \cdot (U_0 - i_1 \cdot (R_1 + R_2) + \frac{(u_C + i_1 \cdot R_2 - d)}{(R_2 + k_{high})} \cdot R_2 - u_C)$$

$$\frac{du_C}{dt} = \frac{1}{C} \cdot (i_1 - \frac{(u_C + i_1 \cdot R_2 - d)}{(R_2 + k_{high})})$$

Now the Euler solver has to be applied, which e.g. for the voltage equation in case ② gives

$$u_{C_{n+1}} = u_{C_n} + \frac{\Delta t}{C} \cdot \left[i_{1_n} - \left(\frac{\left(u_{C_n} + i_{1_n} * R_2 - d\right)}{(R_2 + b)} \right) \right]$$

2.2 Phase change Locking - Conducting

The event handling with the interpolated Shockley diode is similar to the event handling with the ideal diode. In the locking phase the diode voltage u_D corresponds to the state event condition function of the conduction phase. In analogy, the diode voltage u_D in conduction phase is the state event condition function for the locking phase.

The interpolated Shockley diode simplifies the state event recognition, because there is also a voltage drop along the diode when it is forward biased. But in the conducting phase, there are two different sets of equations for calculating i_1, u_C, i_D and u_D depending on the value of the u_D in the former timestep.

The start of conduction phase (SCP) is triggered by the value of u_D in the locking phase. A transition of the value of u_D from positive to a negative number indicates the conducting phase start:

$$u_D \overset{!\,-}{=} i_1 * R_2 + u_C$$

The start of the locking phase (SLP) is triggered by the value of u_D in the conducting phase. A transition of the value of u_D from negative to a positive number indicates the locking phase start:

$$U_D \overset{!\,+}{=} \begin{cases} k_{low} \cdot I_d & 0 \leq u_D \leq -0{,}33 \\ k_{high} \cdot I_d + d & 0{,}33 < u_D \leq 1 \end{cases}$$

So, the change of the linear relation for the diode current is an additional event, which should be treated properly.

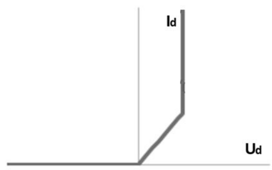

Figure 9: 'Real size' of diode characteristics.

It must be mentioned, that with the diode data given, the exponential function has a very big slope, as well as the linear functions (Figure 9). So also alternatives are possible, e.g. linearisation (see Section 2.5)

	A	B	C	D	E	F	G	H
25	t	U0	i1	Uc	Ud	Id		
35	1,125E-07	0,10080814	7,7192E-05	3,04E-05	1,57E-03	0		
36	0,000000125	0,11196436	8,8126E-05	4,00E-05	1,80E-03	0		
37	1,375E-07	0,1231065	=WENN(UND(E35>0;E36>0);C36+1/$B$6*(B36-C36*($B$4+$B$5)-D36)*$E$6;WENN(E36>=-0,					
38	0,00000015	0,13423313	333;C36+1/$B$6*(B36-$B$4*C36-$B$5*(C36-((D36+C36*$B$5)/($B$5+$B$17)))-D36)*$E$6;C36+					
39	1,625E-07	0,14534287	1/$B$6*(B36-$B$4*C36-$B$5*(C36-((D36+C36*$B$5-$B$19)/($B$5+$B$18)))-D36)*$E$6))					
40	0,000000175	0,15643431	WENN(Wahrheitstest; [Wert_wenn_wahr]; [Wert_wenn_falsch])	9,23E-05	2,73E-03	0,00E+00		

Table 5: Conditional computation of current and voltage depending on phase, interpolated Shockley diode model.

	A	B	C	D	E	F	G	H
25	t	U0	i1	Uc	Ud	Id		
35	1,125E-07	0,10080814	7,7192E-05	3,04E-05	1,57E-03	0		
36	0,000000125	0,11196436	8,8126E-05	4,00E-05	1,80E-03	0		
37	1,375E-07	0,1231065	9,9056E-05	5,10E-05	=WENN(UND(E35>0;E36>0);C37*$B$5+D37;WENN(E36>=-0,			
38	0,00000015	0,13423313	0,00010998	6,34E-05	333;$B$17*F37;$B$18*F37+$B$19))			
39	1,625E-07	0,14534287	0,00012089	7,72E-05	WENN(Wahrheitstest; [Wert_wenn_wahr]; [Wert_wenn_falsch])			

	A	B	C	D	E	F	G	H
25	t	U0	i1	Uc	Ud	Id		
35	1,125E-07	0,10080814	7,7192E-05	3,04E-05	1,57E-03	0		
36	0,000000125	0,11196436	8,8126E-05	4,00E-05	1,80E-03	0		
37	1,375E-07	0,1231065	9,9056E-05	5,10E-05	2,03E-03	=WENN(UND(E35>0;E36>0);0;WENN(		
38	0,00000015	0,13423313	0,00010998	6,34E-05	2,26E-03	E36>=-0,333;(D37+C37*$B$5)/($B$5+		
39	1,625E-07	0,14534287	0,00012089	7,72E-05	2,49E-03	$B$17);(D37+C37*$B$5-$B$19)/($B$5+		
40	0,000000175	0,15643431	0,00013178	9,23E-05	2,73E-03	$B$18)))		
41	1,875E-07	0,16750606	0,00014266	1,09E-04	2,96E-03	WENN(Wahrheitstest; [Wert_wenn_wahr]; [Wert_wenn_falsch])		

Table 6: Conditional computation of diode current and diode voltage depending on interpolation formula.

2.3 EXCEL Implementation – interpolated Shockley diode

The top of the spreadsheet for the model with the interpolated Shockley Diode is almost equivalent to the top of the model with ideal diode (Table 2). Only a few parameters for the linear interpolation for the diode voltage – current relation are added.

Also the simulation of the interpolated Shockley diode model starts in locking phase – for 10 rows without conditions for phase change

In Table 5, cell C37 shows the Euler solver update for the conductor current i_{1_k}, with conditions for phase changes and interpolation selection.

The first distinction is for determining locking or conducting phase, the second one is programmed inside the conducting phase and decides about the linear interpolation function of diode voltage u_{D_k} and diode current i_{D_k} (shown in Table 6, cell E37 and cell F37).

From cell E36 to cell E37 the code changes from **`E36=C36*$B$5+D36`** to

```
E37=WENN(UND(E35>0;E36>0);C37*$B$5+D37;
    WENN(E36>=-0,333;$B$17*F37;
    $B$18*F37+$B$19))
```

2.4 EXCEL simulation interpolated Shockley diode model - results

The smoothening dynamics of the (interpolated) Shockley diode model should eliminate the jumping characteristics of the ideal diode model. The results for capacitor voltage u_C , diode voltage u_D and diode current i_D show the expected smoothed behaviour.

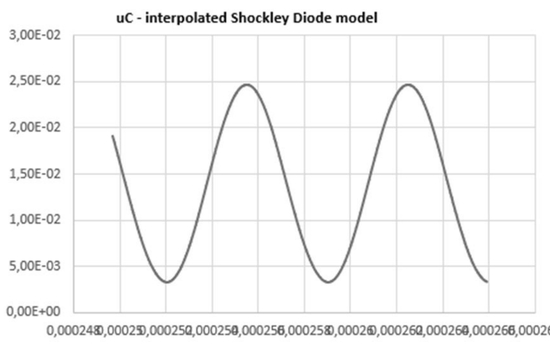

Figure 10: EXCEL simulation results for interpolated Shockley model – capacitor voltage.

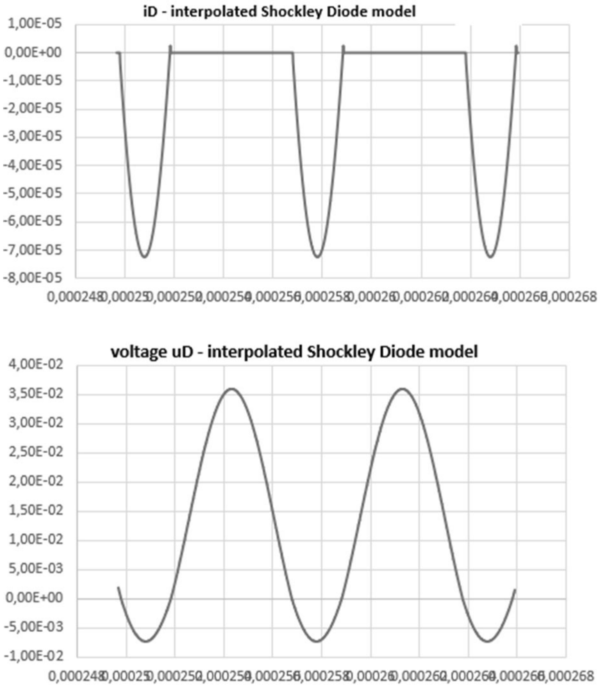

Figure 11: EXCEL simulation results for interpolated Shockley model – diode voltage and diode current.

Diode voltage u_D (Figure 11, below) shows now continuous behaviour, and diode current i_D shows almost no 'spikes', and the resulting capacitor voltage u_C looks now also continuous (Figure 10) – in comparison with results from ideal diode model (Figure 6).

2.5 Linearisation of Shockley formula

A classical approximation for the nonlinear Shockley formula is the linearisation of the exponential function, e.g. at linearisation point $u_{D_{lin}} = 0$,

$$e^{\frac{u_D}{U_T}} = \sum_0^\infty \left(\frac{u_D}{U_T}\right)^n \cdot \frac{1}{n!} = 1 + \frac{u_D}{U_T} + \left(\frac{u_D}{U_T}\right)^2 \cdot \frac{1}{2} + \left(\frac{u_D}{U_T}\right)^3 \cdot \frac{1}{6} + \cdots$$

so that i_D gets the linear approximation formula

$$i_D = I_S \cdot \left(e^{\frac{U_D}{U_T}} - 1\right) \sim I_s \cdot \left(1 + \frac{u_D}{U_T} - 1\right) = \frac{I_s}{U_T} \cdot u_D$$

$$i_D = \frac{I_s}{U_T} \cdot u_D = \frac{1}{k_{lin}} \cdot u_D$$

Now starting again with Kirchhoff's joint formula,

$$0 = -u_C - (i_1 - i_D) \cdot R_2 + u_D$$

we substitute i_D by the above linearised formulation

$$0 = -u_C - (i_1 - \frac{I_s}{U_T} \cdot u_D) \cdot R_2 + u_D$$

and get linear formulas for u_D and i_D:

$$u_D = U_T \cdot \frac{U_C + R_2 \cdot i_1}{I_s R_2 + U_T}, \quad i_D = I_s \cdot \frac{U_C + R_2 \cdot i_1}{I_s R_2 + U_T}$$

These relations are equivalent to the formula for the first region of the Shockley formula interpolation with $u_D = k_{low} \cdot i_d \sim k_{lin} \cdot i_d$:

$$i_D = \frac{1}{k_{lin}} \cdot u_D \rightarrow i_D = \frac{(u_C + i_1 \cdot R_2)}{(R_2 + k_{lin})}$$

Figure 12 shows the situation: instead of the linear function with slope k_{low} now the tangent with slope k_{lin} can be used – and interestingly, the simulations show almost the same results.

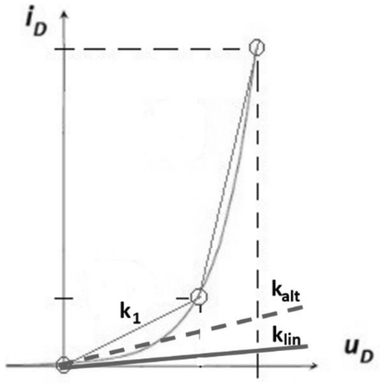

Figure 12: Shockley diode characteristics – interpolated, linearized and approximated around zero.

And interestingly, the indeed same results are produced with a slope k_{alt}, with $k_{lin} < k_{alt} < k_1$, as some experiments have shown. Reason is, that the current i_D is very low, so that approximation near to $u_D = 0$ is sufficient.

Generally, one could also use a linearisation around a proper linearisation point $u_{D_{lin}} \neq 0$:

$$e^{\frac{u_D}{U_T}} = \sum_0^\infty \left(\frac{(u_D - u_{D_{lin}})}{U_T}\right)^n \cdot \frac{1}{n!} \sim 1 + \frac{u_D - u_{D_{lin}}}{U_T}$$

so that u_D becomes

$$u_D = \frac{U_T \cdot (u_C - R_2 \cdot i_1) + R_2 \cdot I_s \cdot u_{D_{lin}}}{R_2 I_s + I_s}$$

There is a tricky algorithmic choice for the 'unknown' $u_{D_{lin}}$: if the formula is used in the course of the Euler solver, then $u_{D_{lin}}$ can be estimated with the previous value $u_{D_{n-1}}$ for the diode voltage:

$$u_{D_n} = \frac{U_T \cdot (u_{C_n} - R_2 \cdot i_{1_n}) + R_2 \cdot I_s \cdot u_{D_{n-1}}}{R_2 I_s + I_s}$$

3 RLC Model – Shockley Diode

The Schockley diode model constitutes an exponential link between diode current and applied voltage (Figure 7). The relation between current i_D and voltage u_D in a Shockley diode follows now a continuous function:

$$i(u_D) = \begin{cases} 0, \; u_D > 0 \\ I_S * (e^{\frac{u_D}{U_T}} - 1), u_D \leq 0 \end{cases}$$

In the conducting phase, there is an exponential-shaped (nonlinear) relation between diode voltage and diode current, which cannot be resolved directly.

3.1 RLC Model with Shockley formula

RLC modelling for the conducting phase first yields

$$\frac{du_C}{dt} = \frac{1}{C} \cdot (i_1 - i_D)$$

$$\frac{di_1}{dt} = \frac{1}{L} * (U_0 - i_1 \cdot (R_1 + R_2) + i_D \cdot R_2 - U_C)$$

In both differential equations now i_D must be substitutes by the Shockley formula:

$$\frac{du_C}{dt} = \frac{1}{C} \cdot \left(i_1 - I_S \cdot \left(e^{\frac{U_D}{U_T}} - 1 \right) \right)$$

$$\frac{di_1}{dt} = \frac{1}{L} \cdot \left(U_0 - i_1 \cdot (R_1 + R_2) + I_S \cdot \left(e^{\frac{U_D}{U_T}} - 1 \right) \cdot R_2 - U_C \right)$$

Now also in Kirchhoff's junction formula

$$0 = -u_C - (i_1 - i_D) \cdot R_2 + u_D$$

i_D is substitutes by the Shockley formula:

$$u_D = u_c + \left(i_1 - I_S \cdot \left(e^{\frac{u_D}{U_T}} - 1 \right) \right) \cdot R_2$$

This nonlinear algebraic equation for u_D is added to the above differential equations for u_C and i_1, which would need u_D as further (algebraic) state variable. The result is a DAE system, a differential-algebraic system, which needs special solvers, combining ODE solving and nonlinear equation solving.

A proper alternative is now to derive a differential equation for the diode voltage u_D . Calculating the derivative of the algebraic equation for u_D results in

$$\dot{u}_D = (\dot{u}_C + \dot{i}_1 * R_2) * \left(1 + \frac{I_S * \left(e^{\frac{u_D}{U_T}} \right) * R_2}{U_T} \right)^{-1}$$

Now the derivatives $\dot{u}_C$ and $\dot{i}_1$ are replaced by the differential equations:

$$\dot{u}_D = \left(i_1 - I_S * \left(e^{\frac{u_D}{U_T}} - 1 \right) \right) + \\ + R_2 \cdot \frac{1}{L} \cdot \left(U_0 - i_1 \cdot (R_1 + R_2) + I_S \cdot \left(e^{\frac{u_D}{U_T}} - 1 \right) \cdot R_2 - u_C \right) \cdot \\ \cdot \left(1 + \frac{I_S * \left(e^{\frac{U_D}{U_T}} \right) * R_2}{U_T} \right)^{-1}$$

Now the Euler solver is applied on these three differential equations, for state update, giving

$$i_{1_{n+1}} = i_{1_n} + \Delta t \cdot f(i_{1_n}, u_{C_n}, u_{D_n})$$

$$u_{C_{n+1}} = u_{C_n} + \Delta t \cdot g(i_{1_n}, u_{C_n}, u_{D_n})$$

$$u_{D_{n+1}} = u_{D_n} + \Delta t \cdot h(i_{1_n}, u_{C_n}, u_{D_n})$$

where the function *f*, *g* and *h* represent the right-hand side of the ODE formulas.

3.2 Phase change Locking - Conducting

In the simulation of the fully dynamic Shockley diode the state event recognition is again based on the diode voltage u_D. Finding the state event of locking phase start is facilitated because of negative voltage u_D in conducting phase. Another advantage is now that no further case distinction in conducting phase is needed for the computation of the diode voltage.

The state event condition function of the conducting phase is given by:

$$U_D \overset{!}{=} i_1 * R_2 + U_C$$

whereby u_D is now solution of the differential equation for u_D in resolved form. completes the

3.3 EXCEL implementation – Shockley diode model

The top of the spreadsheet for the model with the interpolated Shockley Diode is almost equivalent to the top of the models before (Table 2). Only a few parameters for the characteristics of the Shockley equation are added, and an initial value for the ODE for the diode voltage.

Also the simulation of the dynamic Shockley diode model starts in locking phase – for 10 rows without conditions for phase change.

Additionally, now an Euler state update for u_{D_k} must be programmed. In Table 7, cell E37 shows the Euler solver update for the diode voltage u_{D_k}, with conditions for phase changes.

	A	B	C	D	E	F	G	H
25	t	U0	i1	Uc	Ud	Id		
32	0,000000075	0,06726894	4,45523E-05	9,61E-06	9,01E-04	0		
33	8,75E-08	0,07845902	5,53737E-05	1,52E-05	1,12E-03	0		
34	0,0000001	0,08963922	6,62679E-05	2,21E-05	1,35E-03	0		
35	1,125E-07	0,10080814	7,71924E-05	3,04E-05	1,57E-03	0		
36	0,000000125	0,11196436	8,81257E-05	4,00E-05	1,80E-03	0		
37	1,375E-07	0,1231065	9,90563E-05	5,10E-05	=WENN(UND(E35>0;E36>0);C37*$B$5+D37;E36+((			
38	0,00000015	0,13423313	0,000109978	6,34E-05	1/$B$7*(C37-$B$8*(EXP(E36/$B$9)-1))+$B$5*1/			
39	1,625E-07	0,14534287	0,000120887	7,72E-05	$B$6*(B37-C37*($B$4+$B$5)+$B$8*(EXP(E36/			
40	0,000000175	0,15643431	0,00013178	9,23E-05	$B$9)-1)*$B$5-D37))*(1+($B$8*EXP(E36/$B$9)*			
41	1,875E-07	0,16750606	0,000142656	1,09E-04	$B$5)/$B$9)^(-1))*$E$6)			
42	0,0000002	0,17855672	0,000153513	1,27E-04	WENN(**Wahrheitstest**; [Wert_wenn_wahr]; [Wert_wenn_falsch]) 3,20E-03	0,00E+00		

Table 7: Conditional computation of diode voltage with Euler step update.

3.4 EXCEL simulation Shockley diode - results

The results for the dynamics Shockley diode model show generally the expected continuous behaviour (Figure 13), comparisons with results from ideal diode model and interpolated Shockley diode model see the next chapter.

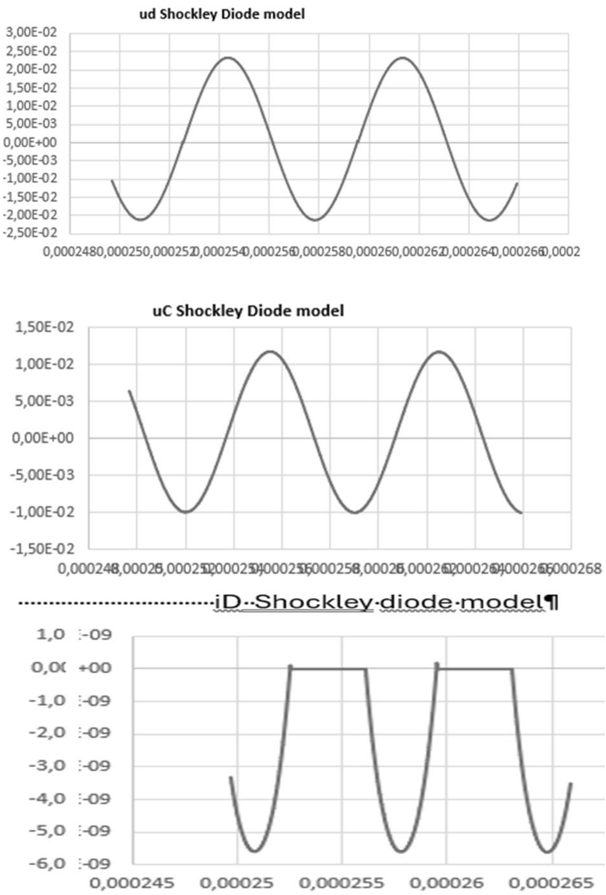

Figure 13: EXCEL simulation results for dynamic Shockley model.

4 Comparison of Results

A comparison of the results is to some extent difficult, as there were some problems with the numeri behaviour of the computation in EXCEL. So observed differences may also be caused by these numerical problems.

In the Figure 14 the diode current for the three different models are shown. The blue graph corresponds to the first case, the idealized diode. Its path through the locking phase is stable, as the diode voltage is at a relatively high level. As the conducting phase is entered, the current rises until its peak value is reached. In the second part of the conducting phase, the current decreases and reaches even positive values.

Per definition an ideal diode is conducting only if it is forward biased, so the computed peaks in positive direction are wrong. The reason therefore is the chosen case distinction which demands two positive values for U_D in a row for switching to the locking phase equation set. The simulation algorithm remains in the equation set for the conducting phase, although it should already have switched.

Analysing the graphs of the two other diode models (interpolated and fully dynamic Shockley diode), the above discussed artefacts do not appear. Case distinction works properly for switching between both phases, although the same conditions are applied. The reason therefore is, that there is a voltage drop along the diode also in conducting phase. This voltage drop is calculated with the diode current at timestep t_k. The voltage remains negative throughout the conducting phase and becomes positive with the current getting positive.

Due to this fact, two timesteps are calculated with the wrong set of equations before it is switched.

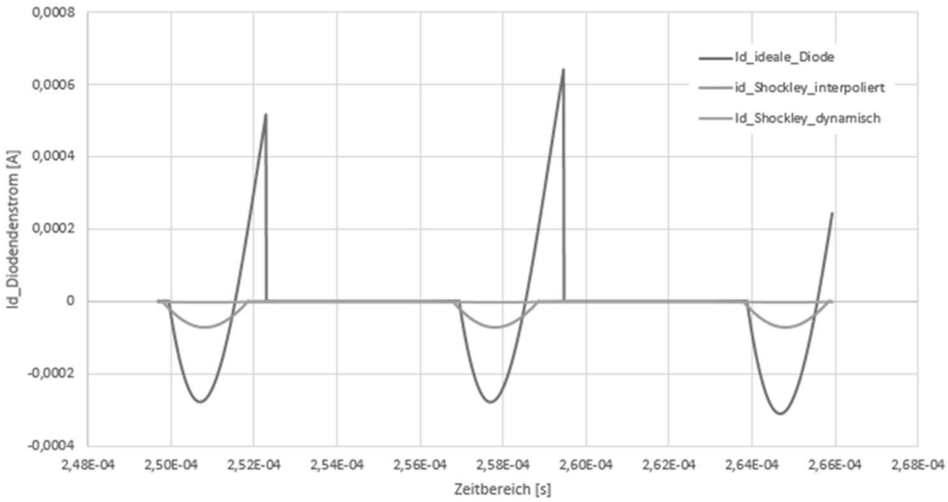

Figure 14: Comparison of simulation results for diode current – ideal diode, interpolated and dynamic Shockley diode.

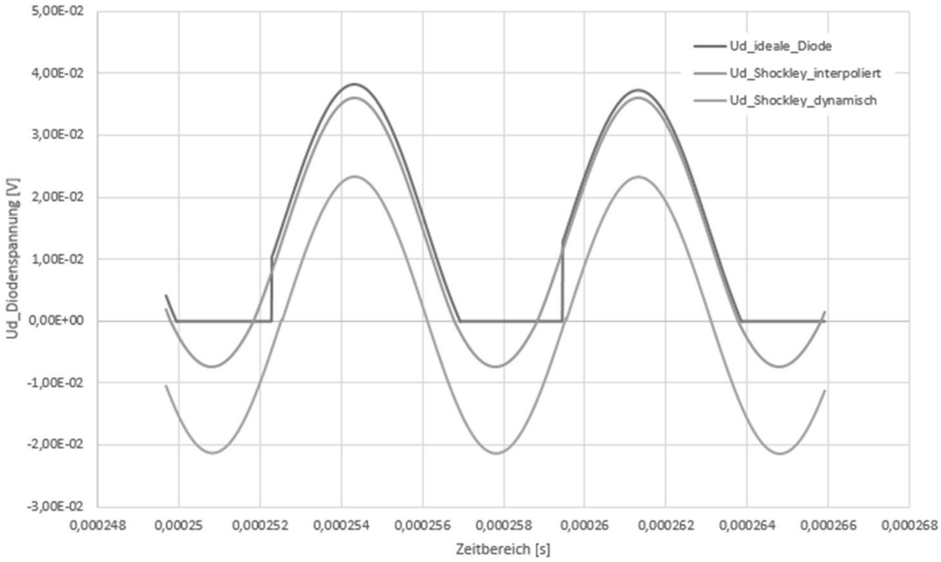

Figure 15: Comparison of simulation results for diode voltage – ideal diode, interpolated and dynamic Shockley diode.

In the ideal diode model at timestep t_k, the diode current is not needed for the computation of the diode voltage which causes the problems mentioned above.

In Figure 15 the diode voltage for the three different models is shown. In the graph, major differences between the three diode models can be observed. The blue line meets the definition of an ideal diode. No voltage drop appears in the conducting phase. Studying the orange graph corresponding to the interpolated Shockley diode, a negative voltage drop in conducting phase can be viewed. The fully dynamic Shockley model shows the same behaviour even more distinctive.

At a closer look on the diode voltage of the ideal model, the case switching problem mentioned above also is observable here. The switching from locking phase to conducting phase works properly, but not the following switch from conducting to locking phase. The equation for the locking phase delivers completely different values compared to the equation of the conducting phase in the timestep before. From one timestep to the following, the diode voltage increases by several orders of magnitude.

References

[1] Körner A, Breitenecker F. State Events and Structural-dynamic Systems: Definition of ARGESIM Benchmark C21. Simulation Notes Europe SNE 26(2), 2016, 117 – 122. DOI: 10.11128/sne.26.bn21.10339