MSFEM for the Linear 2D1D-Problem of Eddy Currents in Thin Iron Sheets

Markus Schöbinger*, Joachim Schöberl, Karl Hollaus

Department of Analysis and Scientific Computing, Vienna University of Technology, Wiedner Hauptstraße 8–10, 1040 Vienna, Austria; *markus.schoebinger@tuwien.ac.at

SNE 30(1), 2020, 39-41, DOI: 10.11128/sne.30.sn.10509
Received: June 10, 2019 (Selected MATHMOD 2018 Vienna Postconf. Publ.), Accepted: December 27, 2019
SNE - Simulation Notes Europe, ARGESIM Publisher Vienna, ISSN Print 2305-9974, Online 2306-0271, www.sne-journal.org

Abstract. A 2D/1D method to simulate the eddy currents in a single thin iron sheet is presented. It utilizes ideas of the multiscale method by decomposing the solution with respect to its dependence on the coordinate directions. Instead of the three dimensional domain, the eddy current problem is solved only on the two dimensional cross section of the sheet, with the behavior of the solution along the thickness being simulated via an expansion into polynomial shape functions. This greatly reduces the number and the coupling density of the degrees of freedom. A numerical example shows a satisfying accuracy for both the $\mathbf{A}$ and the $\mathbf{T}$ formulation.

Introduction

In rotating electrical machines, it is reasonable to assume that each iron sheet is exposed to the same field, thus it suffices to simulate only one sheet. In the case of the thickness of the sheet being small compared to the other dimensions, the three dimensional problem may be further reduced to a two dimensional one, coupled with a separate one dimensional problem in the direction of the thickness, which will be assumed to be the z axis throughout this contribution. Examples of such an approach have been presented in [1] and [4], where the coupling is realized via a nested iteration, and [2], where this principle was used in the context of homogenization.

This contribution presents a novel approach to this idea utilizing a multiscale finite element method (MSFEM, [3]). The main principle is to express the behavior of the solution along the z axis via a polynomial ansatz which directly couples into the two dimensional problem, thereby eliminating the need to repeatedly solve two dependent problems. Such a method will be developed and tested for both the $\mathbf{A}$ formulation and the $\mathbf{T}$ formulation. All models assume a linear, time-harmonic setting.

1 $\mathbf{A}$ Formulation

In three dimensions, the weak form of the eddy current problem is given as: Find the magnetic vector potential $\mathbf{A} \in H(\text{curl})$, satisfying suitable boundary conditions, so that

$$\int_\Omega \mu^{-1} \text{curl}\,\mathbf{A} \cdot \text{curl}\,v + i\omega\sigma\mathbf{A} \cdot \mathbf{v}\, d\Omega = 0 \tag{1}$$

for all test functions $\mathbf{v} \in H(\text{curl})$. In (1) μ denotes the magnetic permeability, i the imaginary unit, ω the angular frequency and σ the electric conductivity.

For the 2D1D model the ansatz

$$\mathbf{A} = \begin{pmatrix} A_{1,1}(x,y)\phi_1(z) \\ A_{1,2}(x,y)\phi_1(z) \\ 0 \end{pmatrix} \tag{2}$$

is chosen. Here the dependency on the coordinate z, aligned with the sheet thickness, is modeled by the linear polynomial function ϕ_1, which is normalized to vary between 1 and -1 along the thickness of the sheet, see Figure 1. $A_{1,1}$ and $A_{1,2}$ stand for the two components of one two dimension unknown $\mathbf{A_1} := (A_{1,1}, A_{1,2})^T \in H(\text{curl})$. Here and in the following the space $H(\text{curl})$ in two dimensions is defined via the two dimensional curl operator, which is given as

$$\text{curl}\,\mathbf{A_1} := \frac{\partial A_{1,2}}{\partial x} - \frac{\partial A_{1,1}}{\partial y}. \tag{3}$$

To derive the 2D problem, the ansatz (2) is used in the three dimensional relation (1) for the trial function and the test function, which leads to

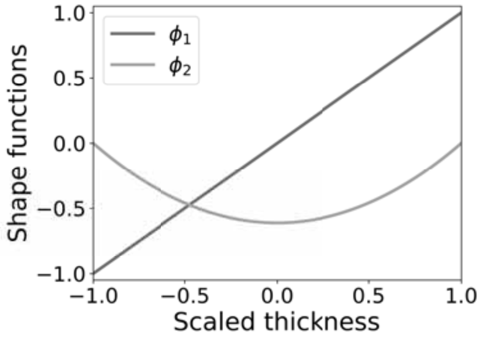

Figure 1: The shape functions ϕ_1 and ϕ_2 on the reference interval $[-1,1]$, which has to be scaled to $[-\frac{d}{2},\frac{d}{2}]$ with the sheet thickness d.

$$\int_\Omega \mu^{-1} \begin{pmatrix} -\phi_1' A_{1,2} \\ \phi_1' A_{1,1} \\ \operatorname{curl} \mathbf{A_1} \end{pmatrix} \cdot \begin{pmatrix} -\phi_1' v_{1,2} \\ \phi_1' v_{1,1} \\ \operatorname{curl} \mathbf{v_1} \end{pmatrix} + i\omega\sigma \begin{pmatrix} A_{1,1}\phi_1 \\ A_{1,2}\phi_1 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} v_{1,1}\phi_1 \\ v_{1,2}\phi_1 \\ 0 \end{pmatrix} d\Omega = 0. \tag{4}$$

Decomposing the iron sheet Ω in the form $\Omega = \Omega_{2D} \times [-\frac{d}{2},\frac{d}{2}]$ with the sheet thickness d, in (4) the integration over the z coordinate can be carried out, using basic analysis for the integrals involving the known function ϕ_1. This results in the two dimensional problem: Find $\mathbf{A_1} \in H(\operatorname{curl})$ so that

$$\int_{\Omega_{2D}} \mu^{-1} \left(\tfrac{4}{d}\mathbf{A_1}\cdot\mathbf{v_1} + \tfrac{d}{3}\operatorname{curl}\mathbf{A_1}\operatorname{curl}\mathbf{v_1}\right) + i\omega\sigma\tfrac{d}{3}\mathbf{A_1}\cdot\mathbf{v_1}\,d\Omega_{2D} = 0 \tag{5}$$

for all $\mathbf{v_1} \in H(\operatorname{curl})$.

Because it is not straightforward to use physically meaningful boundary conditions in this setting, the problem is driven by first solving a corresponding magnetostatic problem, which is then used as a right hand side for (5).

2 T Formulation

For the **T** formulation the three dimensional problem is given as: Find the current vector potential $\mathbf{T} \in H(\operatorname{curl})$ so that

$$\int_\Omega \rho \operatorname{curl}\mathbf{T}\cdot\operatorname{curl}\mathbf{v} + i\omega\mu\mathbf{T}\cdot\mathbf{v}\,d\Omega = 0 \tag{6}$$

for all test functions $\mathbf{v} \in H(\operatorname{curl})$, with given Dirichlet boundary conditions for **T**. Here $\rho = \sigma^{-1}$ denotes the electric resistivity.

For the 2D1D model, a similar ansatz as in the case of the A formulation is chosen:

$$\mathbf{T} = \begin{pmatrix} T_{2,1}(x,y)\phi_2(z) \\ T_{2,2}(x,y)\phi_2(z) \\ 0 \end{pmatrix} \tag{7}$$

Here the behavior in the direction of the thickness is modeled using the even function ϕ_2, which is a quadratic polynomial in z, see Figure 1.

Analogous to the process for the **A** formulation, the ansatz (7) is plugged into (6) and the integration over the z direction is carried out analytically, leading to the problem: Find $\mathbf{T_2} \in H(\operatorname{curl})$ so that

$$\int_{\Omega_{2D}} \mu^{-1} \left(\tfrac{16}{3d}\mathbf{T_2}\cdot\mathbf{v_2} + \tfrac{8d}{15}\operatorname{curl}\mathbf{T_2}\operatorname{curl}\mathbf{v_2}\right) + i\omega\sigma\tfrac{8d}{15}\mathbf{T_2}\cdot\mathbf{v_2}\,d\Omega_{2D} = 0. \tag{8}$$

for all $\mathbf{v_2} \in H(\operatorname{curl})$. The problem is again driven using the solution of an auxiliary problem for the right hand side.

3 A Numerical Example

In order to test the models developed in sections 1 and 2, a simple numerical example is carried out. The dimensions of the problem and the used material parameters can be taken from Figure 2.

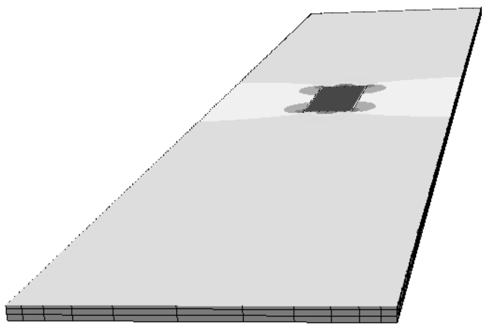

Figure 2: The iron sheet in the numerical example. Its dimensions are a width of $6mm$, a length of $30mm$ and a thickness of $0.5mm$. At the center of the sheet there is a hole of dimension $1.2mm$ times $3mm$. The material parameters are given as $\mu = 1000\mu_0$ and $\sigma = 2.08\times10^6 S/m$.

Figure 3 shows the relative error in the calculated losses. The reference solution was calculated by solving the original problems, (1) and (6), on a three dimensional mesh, respectively. It can be seen that the error increases with higher frequencies, as expected. Out of the given 2D1D models, the one for the **T** formulation performs better, because it is able to simulate the boundary effects, as can be seen in Figures 4 and 5. Further analysis of this problem and how to handle it can be found in [5].

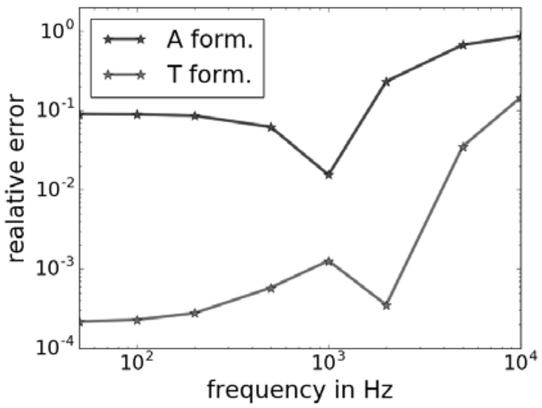

Figure 3: The relative error in the calculated losses for both formulations.

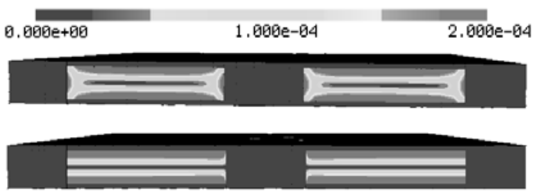

Figure 4: The absolute value of the magnetic vector potential **A** in a cross section of the sheet for the reference solution (top) and the 2D1D model (bottom) at 100Hz. Note that the edge effects are not resolved correctly by the 2D1D method.

4 Conclusion

The presented method allow for a reasonably precise calculation of the eddy current losses for low frequencies. An extension into a higher frequency range is possible by including additional ansatz functions. Future work will include testing the applicability of these models in the nonlinear setting.

Figure 5: The absolute value of curl **T** in a cross section of the sheet for the reference solution (top) and the 2D1D model (bottom) at 100Hz. Note that the edge effects are resolved correctly by the 2D1D method.

Acknowledgement

This work was supported by the Austrian Science Fund(FWF) under Project P 27028 and P 31926.

References

[1] Bottauscio O, Chiampi M. Analysis of Laminated Cores Through a Directly Coupled 2-D/1-D Electromagnetic Field Formulation. *IEEE Trans. Magn.* 2002; 38(5): 2358–2360. doi: 10.1109/TMAG.2002.803573.

[2] Henrotte F, Steentjes S, Hameyer K, Geuzaine C. Pragmatic two-step homogenisation technique for ferromagnetic laminated cores. *IET Sci. Meas. Technol.* 2015; 9(2): 152–159. doi: 10.1049/iet-smt.2014.0201.

[3] Hollaus K, Schöberl J. Some 2-D Multiscale Finite-Element Formulations for the Eddy Current Problem in Iron Laminates. *IEEE Trans. Magn.* 2018; 54(4): 1–4 doi: 10.1109/TMAG.2017.2777395.

[4] Pippuri J, Belahcen A, Dlala E, Arkkio A. Inclusion of Eddy Currents in Laminations in Two-Dimensional Finite Element Analysis. *IEEE Trans. Magn.* 2010; 46(8): 2915–2918. doi: 10.1109/TMAG.2010.2044490.

[5] Schöbinger M, Schöberl J, Hollaus H. Multiscale FEM for the Linear 2-D/1-D Problem of Eddy Currents in Thin Iron Sheets. *IEEE Trans. Magn.* 2019; 55(1): 1–4. doi: 10.1109/TMAG.2018.2879030