Inverse Simulation Methods Applied to Investigations of Actuator Nonlinearities in Ship Steering

David J. Murray-Smith

Emeritus Professor and Honorary Senior Research Fellow, School of Engineering, Rankine Building, University of Glasgow, Glasgow G12 8QQ, Scotland, United Kingdom; *David.Murray-Smith@Glasgow.ac.uk*

Simulation Notes Europe SNE 26(4), 2016, 245 – 256
DOI: 10.11128/sne.26.tn.10356
Received: October 25, 2016
Accepted: December 5, 2016 (Special Issue Review)

Abstract. Actuators associated with control surfaces in aircraft, ships and underwater vehicles often introduce problems in terms of the control characteristics of the vehicle if significant saturation and rate limiting effects are present. Rate limits, in particular, have been linked to a number of well-publicised safety and handling-qualities issues for aircraft. Such limits also present difficulties in ship steering and ship autopilot systems. This paper describes an investigation of the effects of actuator nonlinearities involving a ship steering control application. The method of approach involves the use of inverse simulation to detect the onset of limiting. The paper shows that inverse simulation methods allow direct prediction of situations in which rudder saturation and rate limiting have significant effects in terms of the manoeuvrability of the vessel. It is also shown that a two-stage inverse-simulation method allows direct assessment of the difference between desired and achievable manoeuvres.

Introduction

Inverse dynamic models allow time histories of input variables to be found that permit a given set of output time- history requirements to be achieved. This has relevance for many dynamic problems, especially where actuator performance and limits are important. Inverse models have proved to be particularly useful for investigations involving systems in which a human operator has a central role.

Although analytical approaches to model inversion are of great value, they can present difficulties with many forms of nonlinear model. In recent years, extensive use has been made of simulation techniques for finding inverse solutions rather than depending entirely on analytical methods of inversion. Examples of applications of this kind include aircraft handling qualities investigations and agility studies, both for fixed-wing aircraft and helicopters (see, e.g., [1], [2]). In such cases the inverse solution provides vital information about the relative difficulty of performing different manoeuvres and about control margins available as actuator amplitude or rate limits are approached. In recent years much progress has also been made in using inverse simulation methods in control system design applications (see, e.g., [3], [4]).

1 Models of Actuators and Ship Steering Dynamics

Detailed, physically-based, models of actuators of various kinds are available in the literature and, whether the actuators are hydraulic, electro-hydraulic or electrical in form, the actuator systems have well-defined amplitude and rate limits. Along with the inherent dynamic characteristics of the actuator, these limits are important in determining overall performance of the vehicle or other system within which the actuator is an essential component. For example, actuator performance is of vital importance in aircraft flight control, as discussed in detail by Fielding and Flux [5]. A chronological bibliography of saturating actuators has been prepared by Bernstein and Michel [6] and this includes information from papers and reports involving the use of actuators in many different application areas.

In the case of actuators used for steering in marine vehicles a number of simplified actuator models have been proposed (see, e.g. [7]). Some of these relate directly to earlier work of van Amerongen [8] who, in the context of research on ship steering control systems, proposed the use of a simplified block diagram of the form shown in Figure 1.

This block diagram structure is also used for aeronautical engineering studies of actuator limiting in fixed-wing aircraft and helicopter flight control systems and can be modified quite readily to describe actuators which have second-order characteristics when operating linearly. In this case the block labelled G_r would no longer be a simple gain factor but would have first-order lag characteristics. In principle, accelaration limits as well as rate limits could be incorporated into this type of block diagram structure but this has not been considered in the present investigation. The structure shown in Figure 1, thus represents a general form of model which is capable of describing the linear and nonlinear characteristics of a wide range of actuators in a simple fashion and is appropriate for applications involving marine vehicles or aircraft.

Within the block diagram of Figure 1 the saturation limit block has a simple form and, when the input $G_a\delta_c$ lies in the range between the upper and lower saturation limits (δ_{cU} and δ_{cL}), it behaves as a linear gain element, having unity gain,. However, when the input $G_a\delta_c \geq \delta_{cU}$ the output value is limited at δ_{cU} and, correspondingly, when $G_a\delta_c \leq \delta_{cL}$ the output is limited at δ_{cL}. For many cases of practical importance this limiting behaviour is symmetrical for positive and negative inputs and $\delta_{cU} = -\delta_{cL}$. The rate limit block has an identical form, having unity gain when the output of the block (the rate of change ($\dot{\delta}(t)$) of the actuator ouput position $\delta(t)$) has values that lie within the specified upper and lower actuator rate limits. The rate limit block gives a constant output equal to the positive or negative rate limit when $\dot{\delta}(t)$ has a value beyond the specified upper and lower actuator rate limits.

The type of actuator model outlined above can be used with many different forms of ship model. One nonlinear form of model, which is commonly-used to represent the manoeuvring characteristics of course-stable ships in yaw, is an extended form of Nomoto's first-order model [7], [9] which relates heading variables to the rudder angle.

Although it is based on physical principles, the model involves a number of damping coefficients that must be estimated from data obtained experimentally. It has been shown to be a satisfactory representation for a range of operating conditions [7-9].

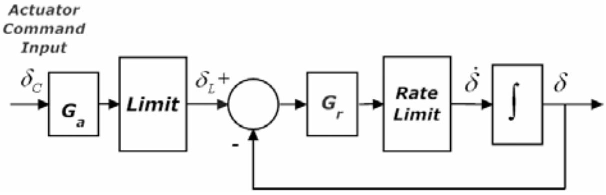


Figure 1: Simplified block diagram of actuator system with amplitude and rate limiting.

The basic model is given by:

$$T\ddot{\psi} + H_N(\dot{\psi}) = K\delta(t) \tag{1}$$

where the variable ψ is the yaw angle (heading) of the vessel, δ is the rudder angle, T is an inertia constant and the function H_N, which is a function of the rate of change of heading $(\dot{\psi})$ is given by:

$$H_N(\dot{\psi}) = n_1\dot{\psi} + n_3(\dot{\psi})^3 \tag{2}$$

where n_1 and n_3 are positive damping constants, known as Norrbin coefficients. For the specific case of the *R.O.V. Zeefakkel*, which is a 45 m long training ship belonging to the Royal Netherland Naval College, the parameters n_1 and n_3 have been estimated for a number of different forward speeds [8]. Combining Eqns (1) and (2) gives:

$$\delta = m\ddot{\psi} + d_1\dot{\psi} + d_3(\dot{\psi})^3 \tag{3}$$

where $m = \frac{T}{K}$, $d_1 = \frac{n_1}{K}$ and $d_3 = \frac{n_3}{K}$. Values of these parameters vary significantly for typical speed values over the range of interest for this vessel, as shown in Table 1.

In this application, the rudder and its associated actuator are modelled using the first-order lag type of description with input saturation and rate limits, as shown in Figure 1. In the linear mode of operation an actuator time constant of 3 s is given by a value of the gain factor G_r of 0.333.

If the required rudder deflection δ_c is the variable subjected to limiting, the gain factor G_a in Figure 1 is unity. For the purposes of this investigation the saturation limit for the rudder is typically of the order of ±35 deg, while the two different rate limit values used in the illustrative examples that follow are ±7 deg/s and ±10 deg/s.

Forward speed (U m/s)	T	K	$m = \frac{T}{K}$	d_1	d_3
2.6	33.0	0.19	173.68	3.3330	3.7037
5	31.0	0.50	62.00	2.0000	0.8000

Table 1: Parameter values used for the model of R.O.V. Zeefakkel [8].

2 Inverse Simulation Methods

Inverse simulation techniques may be divided conventiently into methods that are based on discretised models and are essentially iterative in nature and techniques that are based on continuous system simulation principles. Although the emphasis within this paper is on use of one of the second group of methods, both types of approach are reviewed here since some continuous system simulation approaches have origins in iterative methods involving discretised models.

2.1 Iterative methods of inverse simulation based on discrete models

Several inverse simulation techniques were developed initially for aircraft handling qualities and agility investigations, as mentioned above. The technique that is most widely used was developed first by Hess, Gao and Wang [10] and involves repeated solution of a forward simulation model of the vehicle to allow determination, in an iterative fashion, of inputs that allow the output to follow a specified manoeuvre. This has been termed an 'integration-based' approach. Very similar techniques were developed independently by Thomson and Bradley and their colleagues (see, e.g., [11], [1], [2]). This type of iterative technique is based on the use of gradient methods but search-based optimization methods have also been applied, with success, in a range of applications (see e.g. [12]). Another method, which can be traced back to original work in the aircraft flight mechanics field, involves use of a so-called 'differentiation' method in which a continuous system model of the given system is transformed into a discrete-time description through the use of a finite difference approximation. This approach was developed by Thomson and his colleagues (see, e.g. [13], [14]) in the context of helicopter applications and by Kato and Sugiura [15] for fixed-wing aircraft problems.

Other iterative techniques were also developed for similar applications, including optimization-based approaches by Celi [16] and by Lee and Kim [17]. The paper by Thomson and Bradley [2] provides a useful overview of a number of these iterative techniques, as developed initially for aeronautical applications. Inverse simulation techniques based on discrete forms of model have also been used for the design of model-based output-tracking control systems and a paper by Lu, Murray-Smith and McGookin [3] describes the use of inverse simulation in the design of feed-forward control systems based on a Lynx helicopter model and also for combined steering control and roll stabilisation in a container ship application.

2.2 The continuous system simulation approach

Although the iterative type of approach has been used with considerable success in a number of aeronautical applications, a second (and entirely different) approach to the development of inverse simulation methods has evolved which is based on the use of continuous system simulation principles and avoids the need for iterative solutions.

One approach is based upon the numerical solution of differential algebraic equations (DAEs) (see, e.g. [18], [19]), using DAE solvers. However, it appears that, at present, numerical issues have limited the application of this method to cases involving relatively simple low-order models.

Two other approaches to inverse simulation using continuous system simulation principles are currently available. One of these involves the use of feedback methods (see, e.g., [20], [21]) while the second is based upon an approximate method of differentiation (see, e.g., [22]). From experience gained with other applications, it is known that in the approximate differentiation approach any changes in the structure of the forward model require restructuring of the inverse simulation model and this can be time consuming. In contrast, in the feedback method, changes within the model can be incorporated without changes in the feedback structure (other than possible adjustments of some feedback loop gains). For this reason the feedback approach has been chosen for the work described in this paper.

2.3 Principles of the feedback approach

Some of the earliest developments in inverse simulation involving the use of feedback principles can be found in work carried out at the DLR aeronautical research institute at Braunschweig in Germany, as outlined by Hamel (see e.g. [23]), and discussed in more detail by Gray and von Grünhagen [24] and by Buchholz and von Grünhagen [25]. These methods have more recently been used in a number of applications involving aircraft, process systems and underwater vehicle models (see e.g. [20-21], [26-28]).

A similar type of approach, which is linked specifically to control system design, has been developed by Tagawa and Fukui [29]. Their overall approach is termed 'inverse dynamics compensation via simulation of feedback control systems' (IDCS) and the dervation of an inverse simulation through the use of feedback is a central element of this control design methodology. They have used the IDCS method in control system design applications involving servo-hydraulic actuators and robotics, as described in recent papers [30], [4].

The feedback approach to inverse simulation can best be understood by considering the case of a linear model. The block diagram of Figure 2 involves a single-input single-output linear model $G(s)$ and a feedback loop having a cascaded block with transfer function $K(s)$. The transfer function relating the variable $W(s)$ to a reference input $V(s)$ is given by:

$$\frac{W(s)}{V(s)}=\frac{1}{\frac{1}{K(s)}+G(s)} \tag{4}$$

If the term $1/K(s)$ is very small compared with the magnitude of $G(s)$, over the range of frequencies of interest, the transfer function may be approximated by:

$$\frac{W(s)}{V(s)}\approx\frac{1}{G(s)} \tag{5}$$

Thus, if $K(s)$ is large, the transfer function $W(s)/V(s)$ is a close approximation to the inverse model.

Although a linear single-input single-output system model is used here, the same principles apply to the case of nonlinear models and to multi-input multi-output model structures. While the use of simple high-gain feedback provides acceptable solutions in many cases, it should be noted that the principle of feedback-based model inversion applies also to other feedback structures and the approach is not limited to proportional control methods or to linear models.

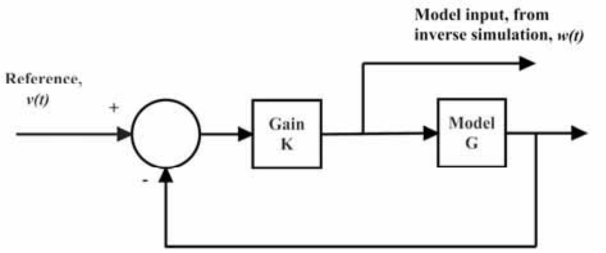


Figure 2: Block diagram for inverse simulation using feedback principles for a given linear or nonlinear model *G*. For a high value of the gain *K*, the variable *w* is a close approximation to the model input required to produce an output that matches a given time history *v*(*t*).

In its origins, the feedback-based approach can be linked back to the use of feedback principles for division and inverse function generation operations in electronic analog computers. Recent work has shown that the approach has very wide applicability [20] and that it allows analysis of the dependence of inverse solutions on parameters of the forward model (without parameter perturbation) through the use of sensitivity models [26]. This can have advantages, especially in the linear case, in terms of the additional physical insight provided when compared with parameter perturbation methods for sensitivity investigation.

One potential problem in applying the feedback-based approach to problems involving actuator saturation and rate limits concerns difficulties arising from possible limit cycle effects. Hard nonlinearities of the type that arise in actuators can give rise to limit cycle phenomena within any feedback loop. For single-input single-output feedback systems, describing function analysis methods (see, e.g. [5], [31]) can be used to predict the existence of limit cycles for feedback systems which involve one dominant nonlinearity and, otherwise, can be described adequately by linear dynamic elements within the feedback loop. The conditions associated with the onset of limit cycle oscillations depend critically on the order of that linearised model and on the form of the nonlinearity. In general, the higher the order of the linear model the more likely it is that limit cycle phenomena will be encountered when saturation or rate limiting effects are present within the feedback loop. Also, nonlinear elements which have describing functions which have a complex form (with imaginary as well as real components) are more likely to give rise to limit cycle oscillations, as discussed by Fielding and Flux [5].

This means that problems of limit cycles are likely to be encountered in attempting to apply the feedback approach to inverse simulation in the case of applications involving significant rate limits. Therefore, in such cases, some modifications to the standard feedback approach may be necessary or entirely different methods of inverse simulation may have to be applied that do not involve the use of feedback.

3 Inverse Simulation Applied to the Ship Model

The first approach considered involves the application of the simple feedback method of inverse simulation, as outlined in Section 2.3 and discussed in greater detail elsewhere (see, e.g. [20], [26], [27]).

3.1 Feedback applied to the ship model with the actuator sub-model included.

Figure 3 is a block diagram which shows the structure of the feedback system which is applied around the ship model, including the actuator sub-model which, in the general case, incorporates saturation and rate limits. The signal used to represent the desired response of the vessel is generated using a reference model. In general terms this must involve a defined output that is consistent with the dynamics of the vessel, with smooth derivatives in order to give realistically smooth actuator control demand movements. In this application the reference input is generated using a third-order reference model which provides appropriate inputs, either in terms of the desired rate of change of heading or the desired heading. In the case involving the desired heading, the structure and parameter values of this reference model are chosen to give a reference signal which rises smoothly from zero to a specified final value of heading over a period of about 30 s. This, together with the corresponding heading-rate reference input, represents appropriate steering dynamics for a vessel of the type being considered. The heading-rate signal from the reference model is used as the reference input in Figure 3.

Feedback was provided by the heading-rate signal which was compared with the heading-rate reference to produce the heading-rate error which was then amplified by the gain *K*.

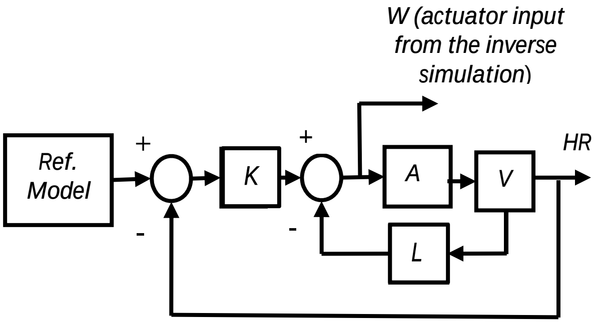


Figure 3: Block diagram of the feedback system used for inverse simulation with the actuator sub-model incorporated within the feedback loop. Here the block *A* represents the actuator and *V* represents the vehicle. The variable *HR* is the vehicle heading rate. The reference model generates the time history of the desired manoeuvre in terms of the required heading-rate time history. The block shown as having a gain factor *L* is a subsidiary feedback loop and, in the case of the application considered here, involves angular acceleration feedback.

As shown in Figure 3, an additional feedback pathway with a gain factor *L* was provided from the heading acceleration signal within ship model as this was found to be beneficial and provided additional damping. Appropriate values for the gain factors *K* and *L* in Figure 3 were determined using basic feedback theory, with some further trial-and-error optimization. A suitable value for the gain factor *K* in the heading-rate feedback loop was found to be 10×10^6 while an appropriate value for the gain factor *L* in the subsidiary loop involving feedback of the angular acceleration was found to be 10×10^4. Results from the inverse simulation studies were found to be relatively insensitive to the precise values used in these feedback loops, provided the two gain factors remained large.

Figures 4-9 show results obtained from the feedback system for a case involving a forward speed of 5 m/s and a demanded heading change of 8 deg. The reference signal is the heading-rate signal obtained from the reference model (Figure 4), corresponding to the heading change shown in Figure 5. The saturation limit in this case is ±35 deg and the rate limits are ±10 deg/s.

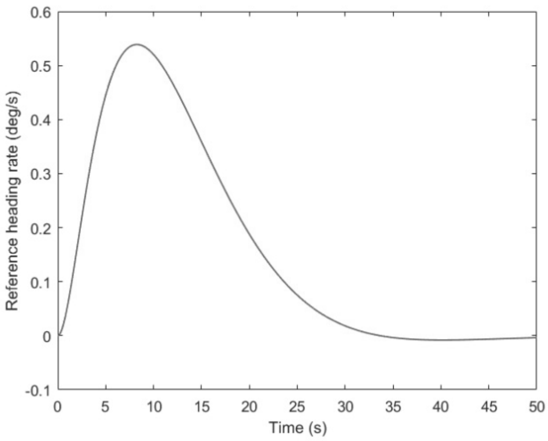
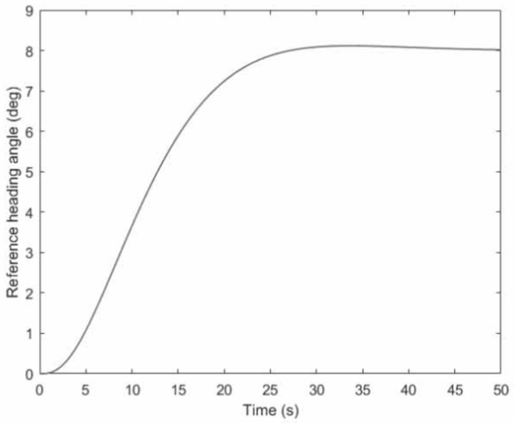


Figure 4: Reference input applied to the feedback system for the case of the ship model with forward speed of 5 m/s and a demanded heading change of 8 deg.

The results in Figures 6 and 7 show that, for the chosen manoeuvre and forward speed condition, the rudder did not approach its angular saturation limit of ±35 deg or its angular rate limit of ±10 deg/s. When applied as input to the forward simulation model, the rudder deflection found from inverse simulation (as shown in Figure 6) produced a heading-rate response which matched almost exactly the required heading rate with heading-rate errors less than $\pm 12\times 10^{-4}$ deg/s over the 50 second response time considered (as shown in Figure 8). This corresponds to a maximum heading error (as shown in Figure 9) of approximately 5.5×10^{-3} deg. Errors in heading angle and heading rate would of course be slightly different for other values of gain factors in the feedback pathways and, in particular, would increase if the gain in the heading-rate feedback loop were reduced significantly. This level of agreement is typical of results found using the feedback method outlined in Section 2, for cases where actuators operate within their limits.

However, if the forward speed of the ship is reduced to 2.6 m/s, the situation changes. At this lower forward speed the manoeuvre is more demanding than that considered in the previous example. Figures 10 and 11 show results for the same 8 deg demanded course change and, it can be seen that the required rudder rate goes well beyond the limit of 10 deg/s, although the rudder deflection does not reach the saturation level of 35 deg.



. **Figure 5:** Heading change corresponding to the heading-rate reference signal of Figure 4.

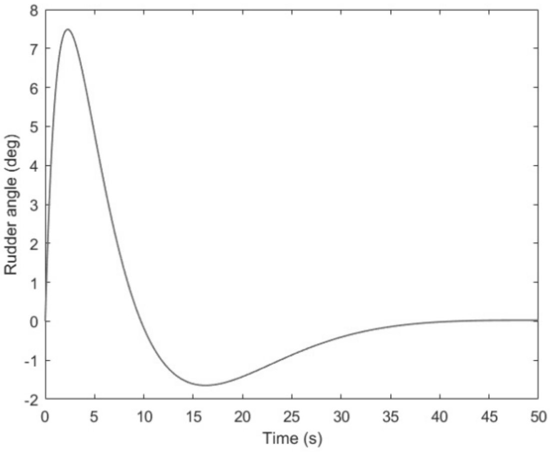


Figure 6: Rudder angle time history found using the inverse simulation process for the ship model with forward speed of 5 m/s.

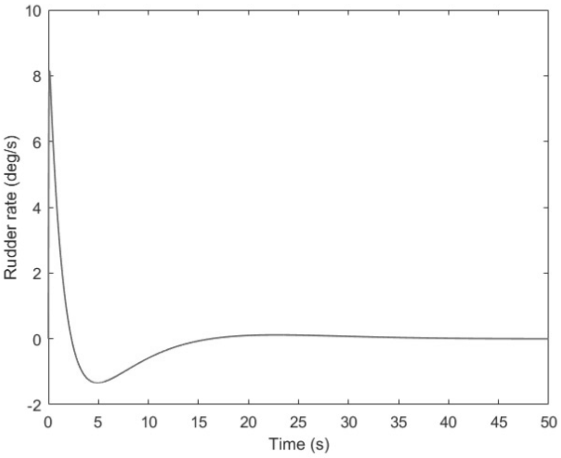


Figure 7: Rudder angular velocity time history found using the inverse simulation process for the ship model with forward speed of 5 m/s.

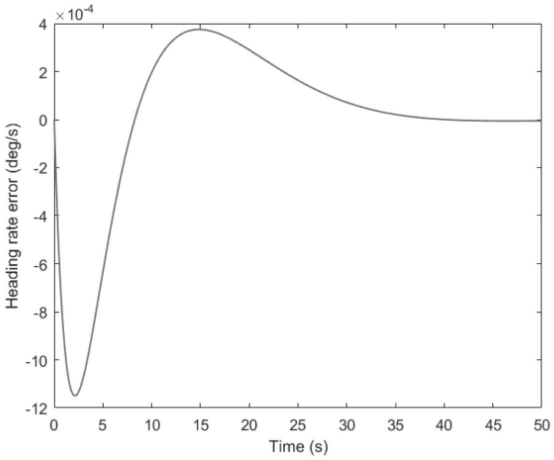


Figure 8: The difference between the heading-rate reference input and the heading-rate found from a forward simulation using the rudder deflection time history of Figure 6.

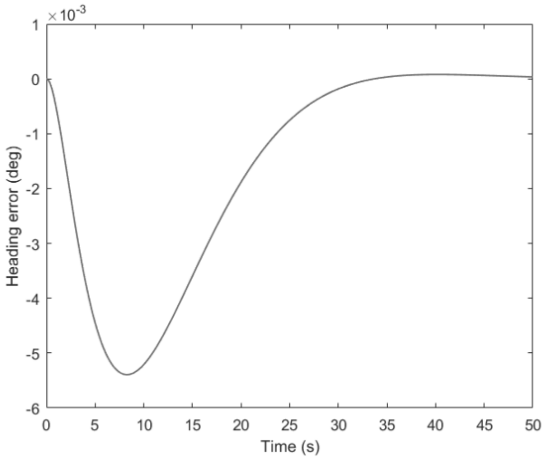


Figure 9: The error in heading corresponding to the results shown in Figure 8.

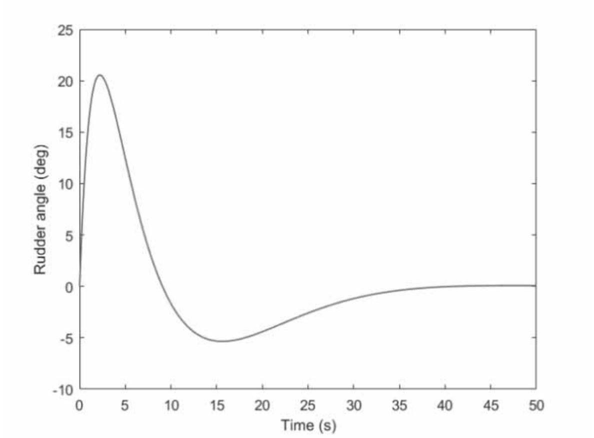


Figure 10: Rudder angle time history found for forward speed of 2.6 m/s. Other conditions for this simulation are the same as for the previous results.

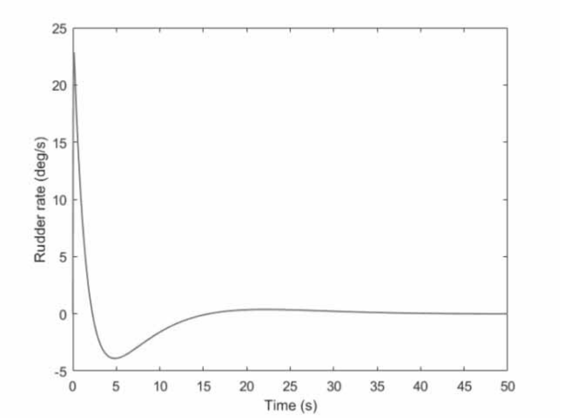


Figure 11: Rudder angular-rate time history found for forward speed of 2.6 m/s. Other conditions for this simulation are the same as for the previous results.

If the demanded heading change is now made larger for the forward speed of 5 m/s, the saturation and rate limits both become important. Figures 12 and 13 show results for a forward speed of 5 m/s and a desired manoeuvre involving a final course change of 40 deg. Clearly the actuator position (rudder angle) now exceeds the 35 deg saturation limit and the angular velocity also exceeeds the 10 deg/s rate limit. This procedure gives a clear indication of situations where demanded manoeuvres exceed the hard limits of the actuator and could cause problems in terms of the control characteristics of the vessel. Thus, if the purpose of the investigation is to establish whether or not a specific manoeuvre gives rise to saturation or rate limiting, the inverse simulation model involving a linear actuator sub-model can provide useful information.

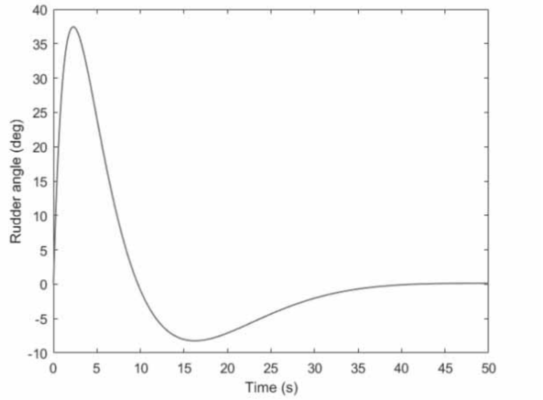


Figure 12: Rudder angle time history found for forward speed of 5 m/s for a demanded manoeuvre corresponding to a 40 deg heading change. Other conditions for this inverse simulation are the same as for the previous results.

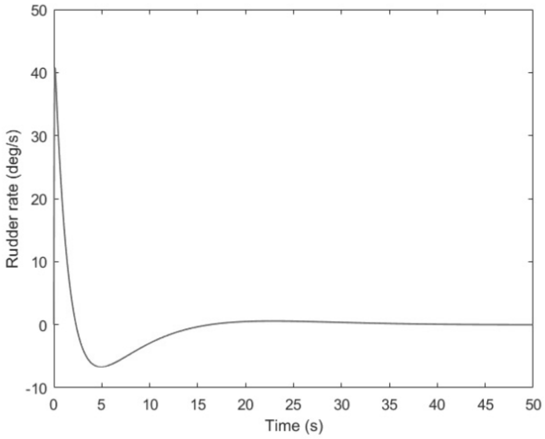


Figure 13: Rudder angular-rate time history found for forward speed of 5 m/s s for a demanded manoeuvre corresponding to a 40 deg heading change.

It can also be concluded that, for less demanding inputs, such as the 8 deg manoeuvre considered in Figures 4-9, inverse simulation methods do provide a direct and clear indication of the margins of control available. In terms of the saturation limit this margin is found from the difference between the maximum rudder deflection and the saturation limit. For the rate limit, the corresponding margin is found by comparing rudder angular velocity values over the complete time history with the rate limit value. In Figure 6 the maximum rudder deflection is about 7.5 deg compared with the saturation limit of 35 deg and there is therefore a large margin of control (27.5 deg of rudder deflection) before the helmsman or autopilot system would encounter problems. Similarly the results of Figure 7 show that the rate of change of rudder angular deflection of about 8 deg/s is below the critical level of 10 deg/s and this suggests that the manoeuvre could be made slightly more demanding before difficulties due to rate limits would be encountered.

The availability of information of this kind is clearly useful in assesing the maneuvrability of a specific vehicle or in considering specific design changes (such as within the actuator and rudder system).

If saturation and rate limits are included within the actuator model, the feedback structure used for inverse simulation changes its behaviour significantly in manoeuvres for which actuator limits are exceeded.

For example, Figure 14 shows the rudder deflection generated from the inverse simulation for a manoeuvre involving a 30 deg change of heading with a forward speed of 2.6 m/s and with a saturation limit of ±35 deg and rate limit of ±7 deg/s. This time history has a very different character from those considered previously and shows a transient which displays limit cycle type oscillations. Although this is not a stable limit cycle phenomenon, investigation based on describing function methods suggests that this transient is an artefact of the feedback methodology and arises as a result of the inclusion of the actuator rate limit. It should be noted that the use of heading feedback rather than heading-rate feedback tends to make this limit cycle behaviour even more pronounced.

In applications where investigation of the effect of actuator limits on the overall dynamic characteristics of the complete vehicle is important, some way must be found of incorporating the nonlinear actuator sub-model within the inverse simulation procedure. In view of the limit cycle problems encountered when the nonlinear actuator sub-model is included within the inverse simulation (as reported above) the simple feedback approach is clearly inappropriate. One possible strategy is outlined in the next section and involves a combination of inverse simulation and conventional forward simulation in a two-stage procedure [28].

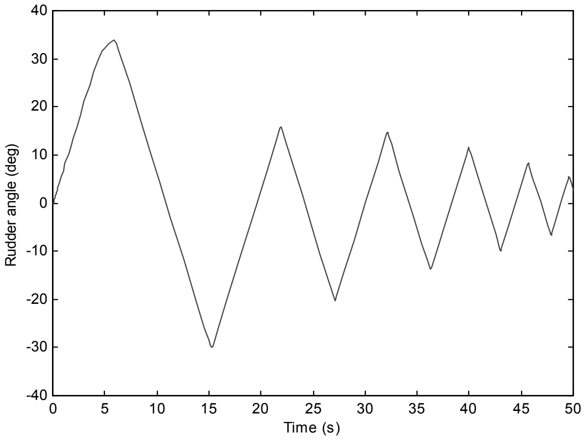


Figure 14: Results in terms of rudder angle obtained by inverse simulation using the feedback approach for the case of the ship model with forward speed of 2.6 m/s and a demanded heading change of 30 deg with a rudder saturation limit of ±35 deg and rudder rate limit of ±7 deg/s.

3.2 A two-stage feedback method

Figure 15 is a block diagram illustrating a two-stage method which allows the feedback approach to be used but which avoids the limit cycle problems encountered with the traditional method in which the nonlinear actuator model is included within the feedback loop. As before, feedback is applied around the ship model to allow a rudder input to be found that produces a heading rate that best matches the reference heading rate. The actuator sub-model is included within the feedback loop, but without the saturation and rate limits. In the first stage of the procedure, inverse simulation based on the feedback structure is used to find an input to this linear actuator model to achieve the desired response if no limits were present. The effect of including the saturation and rate limits is then investigated in the second stage by applying this idealised actuator input found from inverse simulation to a forward simulation of the ship involving the full nonlinear actuator sub-model.

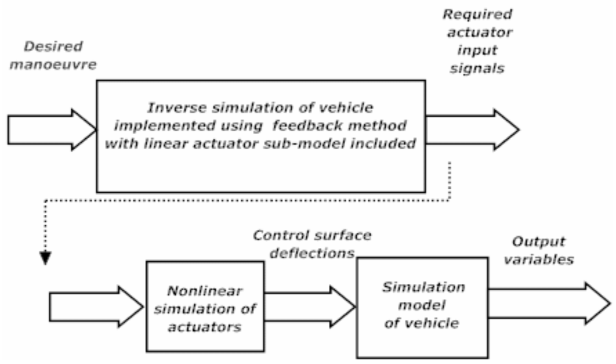


Figure 15: Block diagram of the two-stage procedure for inverse simulation using the feedback method.

Results obtained using this approach are shown in Figure 16 for a case involving a 30 deg heading change for a forward speed of 2.6 m/s with a rudder deflection limit of 35 deg, as before, and a rate limit of ±7 deg/s. The time-history of the rudder response indicates clearly that the rudder moves at the positive rate limit of 7 deg/s for an initial period of about 4 to 5 s, by which time the rudder angle is close to the saturation limit of 35 deg. The rudder then starts to move in the opposite direction and almost immediately reaches the negative rate limit of -7 deg/s. This rate is maintained for a further period of about 6 s, after which the rudder response enters a linear mode of operation, with the rudder angle approaching zero in the final 5 s of the response.

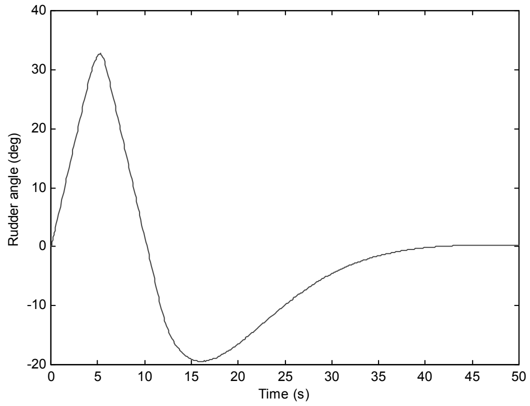


Figure 16: Results in terms of rudder angle obtained by inverse simulation using the two-stage approach for the case of the ship model with forward speed of 2.6 m/s and a demanded heading change of 30 deg with a rudder saturation limit of ±35 deg and rudder rate limit of ±7 deg/s.

Since the primary feedback loop used for the inverse simulation involves comparison of the rate of change of the heading of the vessel with the corresponding quantity from the reference model, it is appropriate to examine the heading-rate error when the rudder deflection time history is used as input to a forward model of the vessel. This is shown in Figure 17 for the 50 s test under consideration. The largest error (about 1.2 deg/s) occurs after about 5s, at the end of the initial period of rudder actuator rate limiting. As would be expected, the heading error found from this forward simulation builds up steadily over the complete time history and reaches almost 35 deg after 50 seconds, as shown in Figure 18.

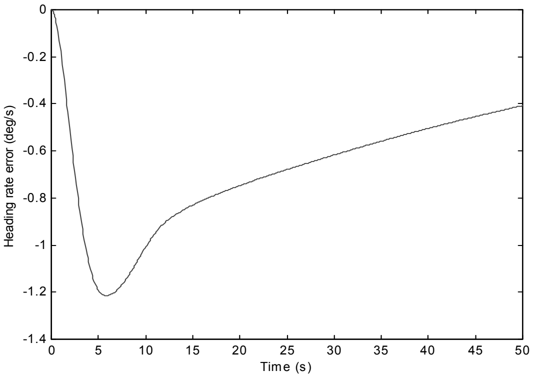


Figure 17: Heading rate error from forward simulation (second stage of the two-stage inverse simulation procedure) using the rudder deflection time history of Figure 16.

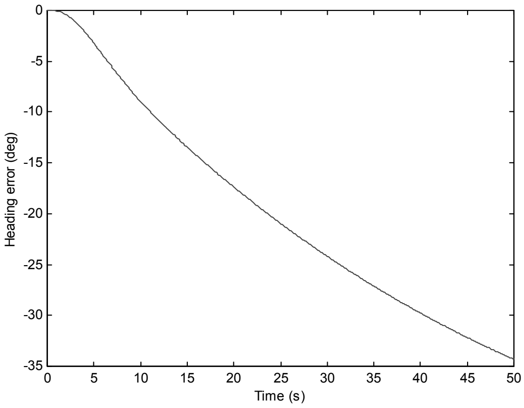


Figure 18: Heading error from forward simulation (second stage of the two-stage inverse simulation procedure) using the rudder deflection time history of Figure 16.

4 Discussion and Conclusions

It can be concluded, from this application, that inverse simulation methods based on feedback principles can provide useful information about the margins of control available before limiting effects in actuators lead to a downgrading of system performance. This is potentially very important in systems involving manual control where actuator input saturation and rate limits can give rise to undesirable oscillatory phenomena such as the pilot-induced oscillations that have been observed in aircraft flight testing. Knowledge of conditions associated with the onset of actuator saturation and rate limiting is also important for the design of automatic control systems, as has been discussed previously in the context of ship control (see, e.g. [7]) and aircraft flight control (see, e.g., [5]).

Inverse simulation techniques are particularly important in all of the above areas because they allow information to be gathered directly about how the input that is needed to perform a specific manoeuvre is affected by the operating condition and parameters of the model.

In order to investigate the effects of actuator saturation and rate limiting on the overall model output, a two-stage inverse-simulation approach has been shown to be useful. This avoids artefacts of the feedback approach which can lead to undesirable limit cycle oscillations. Indeed, it could even be argued that the feedback method of inverse simulation ceases to be valid when hard limiting occurs since the feedback loop then becomes transiently inactive.

However, in most practical situations involving hard limiting of actuators, we are concerned primarily with detecting conditions when limiting occurs and with finding ways of avoiding these, rather than obtaining a complete time-history of the outputs from the inverse simulation model.

It should be noted that the conventional single-stage feedback method of inverse simulation still has practical value for cases in which rate limiting does not occur. This allows inverse simulation to be used as a general-purpose design tool and can assist the designer in investigating the performance of different planned configurations at an early stage in the design process. For example, it can provide answers to questions about the capability of the vehicle under investigation, with known power and control limits, to perform a specified manoeuvre. If it is found that the manoeuvre cannot be carried out inverse simulation may help the designer to make configurational changes, such as a change of actuator characteristics or rudder area that then allow the design requirements to be satisfied.

In general terms, it can be concluded from this application that looking directly at inputs required to perform specific manoeuvres can provide insight that is significantly different from that available using conventional forward simulation tools. The fact that inverse simulation allows the sensitivity of the required input to changes of model parameters to be investigated directly is an important benefit in terms of the design process.

In terms of the specific results obtained in this application, further adjustment of the gain factors in the feedback pathways could be considered and could further improve the accuracy of the inverse simulation results. However, as always, a compromise has to be found between accuracy and computational speed and convenience.

It should be noted that the design of a feedback system for inverse simulation is significantly different from the design of a feedback control system involving a plant model of equivalent complexity. Issues of the robustness of the feedback system in terms of its response to external disturbances, measurement noise and parametric uncertainties, do not have to be considered. This means that less-robust design methods that might be considered inappropriate for control system applications, such as high gain solutions or eigenstructure design methods, can often prove useful in the development of inverse simulations.

References

[1] Thomson DG, Bradley R. The principles and practical application of helicopter inverse simulation. *Simulation Practice and Theory*. 1998; 6 (1): 47-70. doi:10.1016/S0928-4869(97)00012-8.

[2] Thomson D, Bradley R. Inverse simulation as a tool for flight dynamics research - Principles and applications. *Progress in Aerospace Sciences*. 2006; 42 (3): 174-210. doi:10.1016/j.paerosci.2006.07.002.

[3] Lu L, Murray-Smith DJ, McGookin, EW. Investigation of inverse simulation for design of feedforward controllers. *Mathematical and Computer Modelling of Dynamical Systems.* 2007; 13 (5): 437-454. doi:10.1080/ 13873950701344023.

[4] Tagawa Y, Tu JY, Stoten DP. Inverse dynamics compensation via 'simulation of feedback control systems'. *Proc Inst. Mech Eng, Part I: Journal of Systems and Control Engineering*. 2012; 225: 137-153. doi: 10.1243/09596518JSCE1050.

[5] Fielding C, Flux PK. Nonlinearities in flight control systems. *Aeronautical Journal*. 2003; 107(1077): 673-686. doi: 10.1017/S0001924000013543

[6] Bernstein DS, Michel AN., A chronological bibliography of saturating actuators. *International Journal of Robust and Nonlinear Control*. 1995; 5: 375-380. doi: 10.1002/rnc.459005050.

[7] Fossen, TI. *Guidance and Control of Ocean Vehicles*, Chichester, UK: Wiley; 1994. 494 pp.

[8] van Amerongen, J. *Adaptive Steering of Ships: A model-reference approach to improved manoeuvring and economical course keeping.* [PhD Dissertation], Delft University of Technology, The Netherlands; 1982. uuid:2edf96d7-ed7f-40a5-a927-72ff62dea72e.

[9] Kocijan J, Murray-Smith DJ. Robust nonlinear control for ship steering. In Mastorakis NE, editor, *Progress in Simulation, Modeling, Analysis and Synthesis of Modern Electrical and Electronic Systems*, Danvers, Mass. USA, World Scientific and Engineering Society Press, 1999, 235-240.

[10] Hess RA, Gao C, Wang SH. A generalized technique for inverse simulation applied to aircraft maneuvers. *AIAA Journal of Guidance Control and Dynamics*. 1991; 14 (5): 920-926. doi: 10.2514/3.20732.

[11] Rutherford S, Thomson DG. Improved methodology for inverse simulation. *Aeronautical Journal*. 1996; 100 (993): 79-86. doi:10.1017/S0001924000067348 .

[12] Lu L, Murray-Smith DJ, Thomson DG. Issues of numerical accuracy and stability in inverse simulation. *Simulation Modelling Practice and Theory*. 2008; 16(9): 1350-1364. doi: http://dx.doi.org/10.1016/j.simpat.2008.07.003

[13] Thomson DG, Bradley R. An analytical method for quantifying helicopter agility. In: *12th European Rotorcraft Forum, Garmisch-Partenkirchen, Federal Republic of Germany, September 1986*. Paper 45.

[14] Thomson DG, Bradley R. Recent developments in the calculation of inverse solutions of the helicopter equations of motion. In: UK Simulation Council Conference, University College of North Wales, 9-11 September 1987, Ghent, Belgium: UKSC, 1987, 227-234.

[15] Kato O, Sugiura I. An interpretation of airplane motion and control as inverse problems. *AIAA Journal of Guidance, Control and Dynamics.* 1986; 9 (2): 198-204. doi: 10.2514/3.20090.

[16] Celi R. Optimisation based inverse simulation of a helicopter slalom maneuver. *AIAA Journal of Guidance Control and Dynamics.* 2000; 23 (2): 289-297. doi: 10.2514/2.4521

[17] Lee S, Kim Y. Time domain finite element method for inverse problem of aircraft maneuvers. *AIAA Journal of Guidance, Control and Dynamics.* 1997; 20 (1): 97-103. doi: 10.2514/2.4000

[18] Blajer W, Kołodziejczky K. Improved DAE formulation for inverse dynamics simulation of cranes. *Multibody System Dynamics*. 2011; 25 (2): 131-143. doi:10.1007/s11044-010-9227-6.

[19] Thümme M, Looye G, Kurze M, Otter M, Bals J. Nonlinear inverse models for control. In Schmitz G, editor, *Proceedings of the 4th International Modelica Conference, Hamburg, Germany, 7-8 March 2005*. Linköping, Sweden; Modelica Organisation, 2005. 267-279.

[20] Murray-Smith DJ. Feedback methods for inverse simulation of dynamic models for engineering applications. *Mathematical and Computer Modelling of Dynamical Systems.* 2011; 17 (5): 515-541. doi:10.1080/13873954.2011.584323.

[21] Murray-Smith DJ. *Modelling and Simulation of Integrated Systems in Engineering: Issues of Methodology, Quality, Testing and Application.* Cambridge, UK: Woodhead; 2012. 372 pp.

[22] Murray-Smith DJ. An approximate differentiation method of inverse simulation based on a continuous system simulation approach. *Simulation Notes Europe*. 2013; 23(3-4):111-116. doi:10.11128/sne.23.tn.10201.

[23] Hamel PG. Aerospace vehicle modelling requirements for high bandwidth flight control. In Cook MV, Rycroft MJ, editors, *Aerospace Vehicle Dynamics and Control*, Oxford, UK; Clarendon Press, 1994. 1-31

[24] Gray GJ, von Grünhagen W. An investigation of open-loop and inverse simulation as nonlinear model validation tools for helicopter flight mechanics. *Math. and Comp. Modelling of Dynamical Syst.,* 1998; 4: 32-57.

doi: 10.1080/13873959808837067.

[25] Buchholz JJ, von Grünhagen, W. *Inversion Impossible?* Technical Report, University of Applied Sciences Bremen, Germany, 2004.

[26] Murray-Smith DJ. The application of parameter sensitivity analysis methods to inverse simulation models, *Mathematical and Computer Modelling of Dynamical Systems*, 2013; 19 (1): 67-90.

doi:10.1080/13873954.2012.696271.

[27] Murray-Smith DJ. Inverse simulation and analysis of underwater vehicle dynamics using feedback principle, *Mathematical and Computer Modelling of Dynamical Systems*, 2014; 20 (1): 45-65.

doi: 10.1080/13873954.2013.805146.

[28] Murray-Smith DJ, McGookin EW. A case study involving continuous system simulation methods of inverse simulation for an unmanned aerial vehicle application. *Proc. Inst Mech Eng, Part G, Journal of Aerospace Engineering.* 2015; 229(14): 2700-2717.

doi: 10.1177/0954410015586842

[29] Tagawa Y, Fukui K. Inverse dynamics calculation of nonlinear model using low sensitivity compensator. In: *Proceedings of Dynamics and Design Conference. 1994*, 185-188.

[30] Venture G, Kojima T, Tagawa Y. Fast motion control of robotic systems using inverse dynamics compensation via ‘simulation of feedback control systems’ (IDCS). In: *Proc. 1st Joint Conf. on Multibody System Dynamics, May 25-27 2010, Lapeenranta, Finland*.

[31] Atherton DP. *Nonlinear Control Engineering*, New York, USA:Van Nostrand Rienhold; 1982. 470 pp.