

System Dynamics for Modelling Emotions: from Laura - Petrarch to Nowadays Couple

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Abstract. F. J. Jones – a scientist for literary – analyzed Petrarch's 'Sonnets to Laura' with respect to positive and negative emotions and recognized in the changes between love and despair an oscillating behaviour, which he called Petrarch's emotional cycle. The mathematician S. Rinaldi investigated this cycle from viewpoint of stability, established a nonlinear ODE model for this oscillating emotions and inspiration.

This contribution makes use of 'System Dynamics' – SD - to set up a model for the emotional relations between a couple, first for the couple Laura–Petrarch, and second, for a nowadays couple, by extending the states of the system.

In principle, emotions and inspiration emerge from a source, and are fading into a sink – best suited for the SD modelling paradigm. The rate variables and the controlling parameters for increase and decrease of emotions create a broad variety of emotional behaviour and of degree of inspiration, and require nonlinear approaches for driving forces for the emotions, especially in the feedback between Laura and Petrarch (in SD nonlinear auxiliary variables).

Using Jones' emotional cycle as data, parameters in the 'Laura-Petrarch Model' can be identified. Interestingly, again the poems help determining at least the qualitative size of some parameters. Changes some of these historic parameters describe nice case studies: Laura's emotions are fading faster, or positive appeal of Petrarch changes emotion behaviour qualitatively.

The Laura-Petrarch SD model allows to develop a 'Nowadays Couple Model' by extending states in the SD model (taking into account the equality of woman and man nowadays). The historic parameters from the 'Laura-Petrarch Model' can be used as basis for parameters in the 'Nowadays Couple' SD model, – allowing some nice simulation experiments for the development of emotions, from ecstatic up and down to never-ending languidness, from attraction to denial, from natural course to course intervention by aesthetic surgery.



Figure 1: Portraits of Laura and Petrarch, from Biblioteca Medici Laurenziana, Ms. Plut. cc. VIIIv- IX, Florence, Italy (courtesy of Ministero per i Beni Culturali e Ambientali), from [2].

Introduction

Francis Petrarch (1304-1374), is the author of the *Canzoniere*, a collection of 366 poems (sonnets, songs, sestinas, ballads, and madrigals). History tells: in Avignon, at the age of 23, he met Laura, a beautiful but married lady (Figure 1); he immediately fell in love with her which lasted longer than Laura's life; although this love was not really reciprocated (he never met her), he addressed more than 200 poems to her over the next 21 years. Indeed, the poems express bouts of ardour and despair, snubs and reconciliations, making Petrarch the most lovesick poet of all time.

F. J. Jones – a scientist for literary – analysed Petrarch's *Sonnets to Laura* with respect to positive and negative emotions and recognised in the changes between love and despair an oscillating behaviour, which he called *Petrarch's emotional cycle* ([1]).

The mathematician S. Rinaldi investigated this cycle and established a nonlinear ODE model for this oscillation, with Laura's emotion, with Petrarch's emotion, and with Petrarch's inspiration as state variables – based on classical mathematical analysis and stability considerations in ODE systems ([2]).

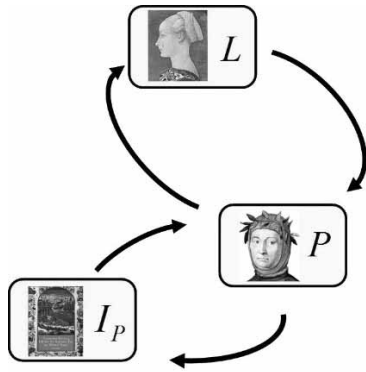


Figure 2: Causal loop diagram between Laura's and Petrarch's emotions and Petrarch's inspiration.

Petrarch's emotions – and Laura's 'counter'-emotions, expressed indirectly in the poems, aggregate over time flowing from sources to a sink, driven by feedback from each other: the key words aggregation, flow, source, sink, feedback almost request for *System Dynamics* as modelling tool for the dynamics of emotion.

System Dynamics (SD) is a well-known modelling and simulation approach, introduced by J. Forrester [5]. The modelling procedure is based on three steps:

- First, a *causal loop diagram* (CLD) visualizes the relations between the aggregated variables qualitatively- Figure 2 shows the principle feedbacks within Laura's emotion, Petrarch's emotion, and Petrarch's inspiration,
- then a stock and flow diagram (SFD) describes the relations quantitatively by flows and feedbacks,
- and additionally the SFD can be directly implemented for computer simulation (nowadays automatically by simulators which understand SD modelling).

Indeed SD turns out to be a good choice as modelling tool (see [3]), but SD is also a good basis for analysis and simulation. On the other side, the principle basic structure of SD with flow and feedback is related to transfer functions, which alternatively can be used as graphical modelling approach for an emotional cycle as discussed in a related contribution [4].

1 Petrarch's Emotional Cycle

Petrarch's poems express 'bouts of ardour and despair', and change drastically in time. Unfortunately, only few lyrics of the *Canzoniere* are dated. In 1995, Frederic Jones presented an interesting approach to the chronological ordering problem of Petrarch's poems in his book *The Structure of Petrarch's Canzoniere* ([1]).

Jones concentrated on Petrarch's poems written at lifetime of Laura (the first, sonnet X, was written in 1330 and the last, sonnet CCXII, in 1347). First, he analysed 23 poems with fairly secure date. After a careful linguistic and lyrical analysis, he assigned grades for the poems, ranging from -1 to +1, establishing *Petrarch's emotional cycle*: the maximum grade (+1) stands for ecstatic love, while very negative grades correspond to deep despair.

The following examples illustrate this grading (in quotations, the English version is taken from an English translation of the *Canzoniere* by Frederic Jones):

- Sonnet LXXVI, great love → grade: +0.6
*Amor con sue promesse lusingando,
mi ricondusse alla prigione antica*
[Love's promises so softly flattering me
have led me back to my old prison's thrall.]
- Sonnet LXXIX, great despair → grade: -0.6
*Così mancando vo di giorno in giorno,
sì chiusamente, ch'i' sol me ne accorgo
et quella che guardando il cor mi strugge.*
[Therefore my strength is ebbing day by day,
which I alone can secretly survey,
and she whose very glance will
scourge my heart.]

In a second step, F. Jones analysed and 'graded' all the other poems with unknown date and checked, in which part of the cycle they could fit. Taking into account additional historical information, he could date these poems.

Displaying the grades over time (Figure 3), F. Jones detected an oscillating behaviour of the grade values, which he called *Petrarch's emotional cycle* $E(t)$, with a period of about four years.

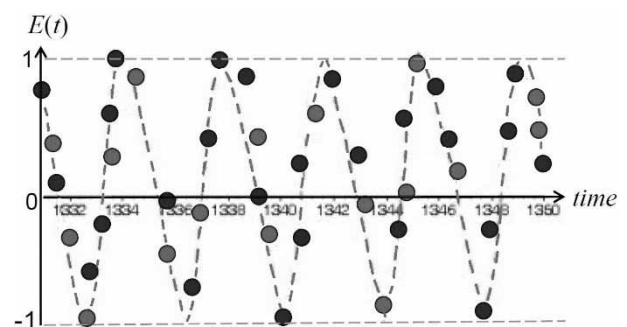


Figure 3: Petrarch's emotional cycle $E(t)$ – dashed line, with 'graded' poems; red dots for securely dated poems, blue dots for poems dated on indirect data (sketch).

2 Model Approach by System Dynamics

System Dynamics (SD) is a well-known modelling approach, introduced by J. Forrester ([5]). SD is a methodology for studying and managing complex feedback systems, such as one finds in business and other social systems. In fact it has been used to address practically every sort of feedback system between aggregated states. Feedback refers to the situation of X affecting Y and Y in turn affecting X e.g. by a chain of causes.

System Dynamics is more than a modelling method – SD diagrams can directly implemented as computer models for simulation experiments.

2.1 Causal loop diagram model

For modelling, SD starts at qualitative level with a causal loop diagram. A *causal loop diagram* (CLD) is a diagram that aids in visualizing how interrelated variables affect one another.

The CLD consists of a set of nodes representing the variables connected together by causal links (Figure 2, causal links between emotions and inspiration in the Laura–Petrarch model). The relationships between these variables, represented by arrows, can be labelled as positive ‘+’ or negative ‘-’. Positive causal links ‘+’ means that the two nodes move in the same direction, i.e. if the node in which the link start increases, the other node also increases. Similarly, if the node in which the link starts decreases, the other node decreases. Negative causal links ‘-’ are links in which the nodes change in opposite directions (an increase causes a decrease in the other node, or a decrease causes an increase in another node). Sometimes the relations are indefinite ‘+-’

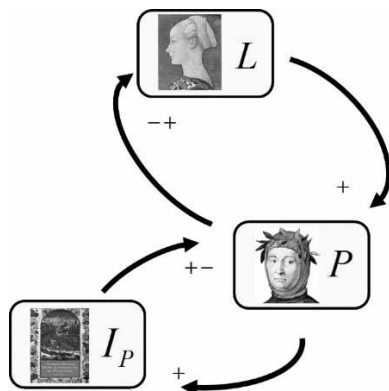


Figure 4: Fully labelled causal loop diagram between Laura's and Petrarch's emotions and Petrarch's inspiration.

In case of Laura's and Petrarch's emotions $L(t)$ and $P(t)$, and Petrarch's inspiration $I_P(t)$, the causal relations are evident and can be partly labelled (Figure 4), but two links are indefinite.

2.2 Stock and flow diagram model

SD continues the modelling process now at the quantitative level by a *stock and flow diagram* (SFD), sometimes also called *level and rate diagram*. Nowadays diverse literature on SD exists, e.g. [6]. A SD model consists of six basic elements summarized in Table 1.







Element	Representation	Description
Stocks (Levels)		Describe the state of the system at each time and represent aggregates
Flows		Describe the changes of the stocks; are basically auxiliaries and are only allowed between stocks and stocks and sinks/sources
Parameters		Are constants and represent rates on which changes of stocks are dependent
Auxiliaries		Are necessary to describe more complex relations combining more inputs
Sink/Source		Describe the boundaries of the system
Links		Describe the causalities of other elements

Table 1: Basic elements of System Dynamics.

A *stock* variable is measured at one specific time. It represents a quantity existing at a given point in time, which may have been accumulated in the past. A *flow* variable is measured over an interval of time. Therefore a flow would be measured *per unit of time*. The variables in the CLD must be identified either as stock (level) or flow (rate) – or as auxiliary, and each stock (level) is connected in the SFD with its inflow – coming from a source- and by its outflow to a sink; flows are represented by double arrows and flow-controlling valves (rates). The causal links from the CLD are found in the SFD as characterising influences from stocks to flows (or from parameters and auxiliaries to flows). SD makes use of auxiliaries to define more complex feedbacks; auxiliaries may have more than one input, and very often they are table functions defined by data.

3 Laura-Petrarch SD Model

For the dynamics of emotion and inspiration under investigation, Laura’s emotion $L(t)$, Petrarch’s emotion $P(t)$, and Petrarch’s inspiration and $I_P(t)$ are considered as stocks. The basic element of the SFD is the behaviour of emotion and inspiration – they are fading when time goes on – they decrease with respect to their intensity (feedback of stock to output flow; Figure 5).



Figure 5: Basic stock and flow diagram for emotions.

Emotions and inspiration are driven by stimulations – input flow of the stock. Petrarch’s emotion $P(t)$ is driven by Laura’s emotion $L(t)$ (feedback from stock Laura to input flow Petrarch; Figure 6), and by his inspiration $I_P(t)$ (feedback from stock Inspiration to input flow Petrarch; Figure 6). Laura’s emotion $L(t)$ is driven by Petrarch’s emotion $P(t)$ (feedback from stock Petrarch to input flow Laura; Figure 6), and the inspiration $I_P(t)$ is driven by Petrarch’s emotion $P(t)$ (feedback from stock Petrarch to input flow Inspiration; Figure 6). So a first simple SFD (Figure 6) for the dynamics shows the feedback structures for the driving stimulations and for the decrease by fading (Figure 6).

SD’s modelling procedures now quantify the qualitative SFD by introducing parameters and auxiliaries for the causal links and for the influences on the flows. Laura’s and Petrarch’s emotions and Petrarch’s intuition are fading with certain celerity, characterised by the gain parameters α_L , α_P , and α_{IP} in the direct feedbacks.

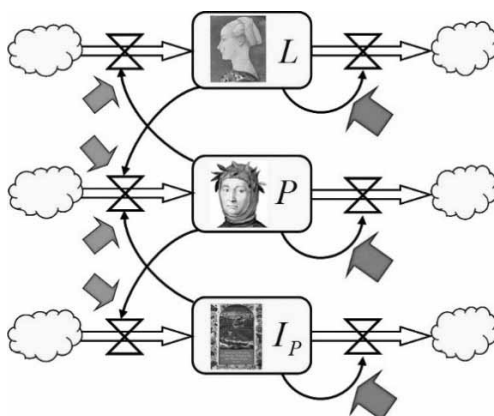


Figure 6: Qualitative stock and flow diagram for Laura’s and Petrarch’s emotions and for Petrarch’s inspiration.

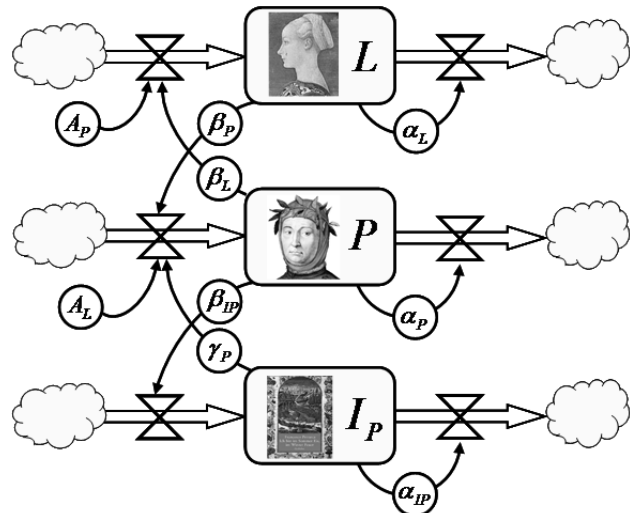


Figure 7: Stock and flow diagram for Laura’s and Petrarch’s emotions and for Petrarch’s inspiration with only linear influences.

The driving stimulations for the emotions $L(t)$ and $P(t)$ are given by a general appeal (A_L Laura’s appeal, A_P Petrarch’s appeal) and by the respective gained ‘counter’-emotion, which is for first a proportion of the respective emotion, characterized by gaining parameters β_P and β_L in the feedbacks: Petrarch’s emotion is stimulated by an additional feedback from inspiration, a feedback from inspiration, characterized by a gaining parameters γ_P in the feedback. The driving stimulation for inspiration $I_P(t)$ is a proportion of Petrarch’s emotion, characterized by gaining parameter β_{IP} in the feedback.

The resulting SFD for the dynamics of emotion and inspiration in Figure 7 shows all basic feedbacks and direct inputs for the flows, gained with parameters or simple ‘gaining’ auxiliaries. This structure is a classic linear one: the ODE system shows only linear behaviour, and is therefore only a first approach for the more complex relations.

But two of the driving stimulations turn out to be more complex, Laura’s reaction on Petrarch’s emotion, and Laura’s appeal to Petrarch – the ‘simple’ gaining with a parameter and the simple constant appeal A_L are not sufficient.

Complexity of Laura’s reaction. First, the reaction feedbacks between the emotions need further investigations. The ‘linear’ approach more or less says, that individuals love to be loved and hate to be hated, and that they expect a ‘linear’ feedback from the other. This ‘simple’ behaviour is evident for Petrarch, since in his poems the poet has very intense reactions to the most relevant signs of antagonism from Laura.

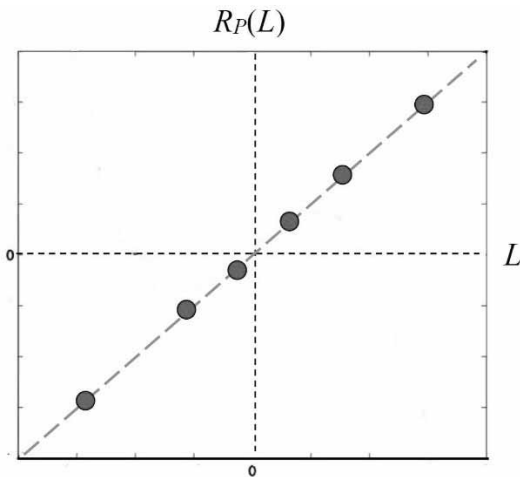


Figure 8: Petrarch's reaction on Laura's emotion
 – red dots tabulated function'
 – dashed line as interpolating linear line.

So indeed this emotion directly follows the emotion shown by Laura, displayed in Figure 8 as table function (red dots with tabulated values, dashed line as interpolating curve), or given in mathematical terms:

$$R_p = \beta_p \cdot L.$$

Laura evolves a more complex reaction to Petrarch's emotion – summarized in Figure 9 (red dots with tabulated values, dashed line as approximation curve). A 'linear' reaction function is not appropriate for Laura, except for little emotions, thus interpreting the natural inclination of a beautiful high-society lady to stimulate harmless flirtations (Figure 9, first blue dots left and right from zero). But Laura never goes too far beyond gestures of pure courtesy: she smiles and glances.

However, when Petrarch becomes more demanding and puts pressure on her, even indirectly when his poems are sung in public, she reacts very promptly and rebuffs him. This is not an assumption, this behaviour can be read more or less explicitly in a number of poems, e.g. in Sonnet XXI:

- In Sonnet XXI, Petrarch claims:
*Mille fiate, o dolce mia guerrera,
 per aver co' begli occhi vostri pace
 v'aggio proferto il cor; ma voi non piace
 mirar si basso colla mente altera.*
 [A thousand times, o my sweet enemy,
 to come to terms with your enchanting eyes
 I've offered you my heart, yet you despise
 aiming so low with mind both proud and free.]

Consequently, the reaction function $R_L(P)$ should, for positive emotions of Petrarch, first increase, and then decrease (Figure 9, blue dots right from zero).

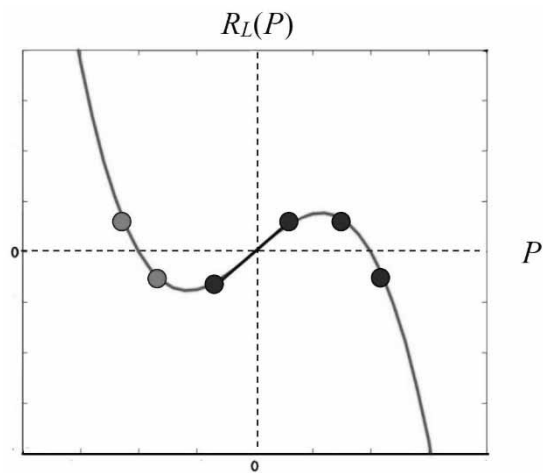


Figure 9: Laura's reaction on Petrarch's emotion
 – red and blue dots: tabulated function'
 – dashed line: approximating curve
 (cubic function).

But Laura's reaction is also more complex ('nonlinear') for Petrarch in despair (negative emotions of Petrarch). Indeed, when the poet despairs, Laura feels very sorry for him, showing despair herself. Following her genuine Catholic ethic she arrives at the point of overcoming her antagonism by strong feelings of pity, thus reversing her reaction to the passion of the poet. This behavioural characteristic of Laura is repeatedly described in the *Canzoniere*, e.g. in Sonnet LXII

- In Sonnet LXIII Petrarch writes:

*Volgendo gli occhi al mio novo colore
 che fa di morte rimemorar la gente,
 pietà vi mosse; onde, benignamente
 salutando, teneste in vita il core.*

[Casting your eyes upon my pallor new,
 which thoughts of death recalls to all mankind,
 pity in you I've stirred; whence, by your kind
 greetings, my heart to life's kept true.]

Consequently, the reaction function $R_L(P)$ should for negative emotions of Petrarch first decrease, and then increase (Figure 9, blue dot and red dots left from zero).

Combining both branches of this the heuristic feedback results first in a table function with few n heuristically derived tabulated values, which directly can be used as auxiliary in SD models:

$$R_L(P) = TAB \left((P_1, R_{L,1}), \dots, (P_n, R_{L,n}); P \right)$$

Mathematically this table function can be approximated by a cubic function (displayed in Figure 9 as dashed curve), which alternatively can be used as auxiliary in the SD model, or for analytical investigations:

$$R_L(P) = P \cdot \left(1 - \left(\frac{P}{\gamma_L}\right)^2\right)$$

Complexity of Laura's appeal. Laura's appeal A_L to Petrarch evolves also complex behaviour, it depends on inspiration $I_P(t)$, which indirectly expresses, whether Petrarch's desire is poetic or passionate. And there is no doubt that the tensions between Petrarch and Laura are of a passionate nature, expressed in Petrarch's work:

- In Sonnet XXII, Petrarch writes:
*Con lei foss'io da che si parte il sole,
 et non ci vedess' altri che le stelle,
 sol una nocte, et mai non fosse l'alba.*
 [Would I were with her when first sets the sun, and no one else could see us but the stars, one night alone, and it were never dawn.]
- And in his *Posteritati*, Petrarch confesses:
*Libidem me prorsus expertem dicere
 posse optarem quidem, sed si dicat mentiar.*
 [I would truly like to say absolutely that I was without libidinousness, but if I said so I would be lying].

These investigations in Petrarch's writing conclude, that Laura's appeal to Petrarch depends on antagonism between lyric inspiration and desire, taking into account the well-established fact that high moral tensions, like those associated with artistic inspiration, attenuate the role of the most basic instincts. The Appeal A_L becomes dependent on the inspiration $I_P(t)$.

Figure 10 sketches the assumed correlation: the less inspiration (and the less time used for writing), the higher the desire lets increase the appeal (Figure 10, left red dots); and the more inspiration (and the more time for poetry), the lower the passionate part in the appeal (Figure 10, right blue dots).

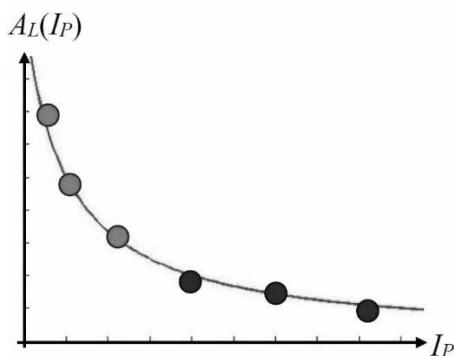


Figure 10: Laura's appeal to Petrarch's dependent inspiration: poetic vs. passionate desire
 - blue and red dots: heuristic tabulated values
 - dashed curve: approximating hyperbolic function.

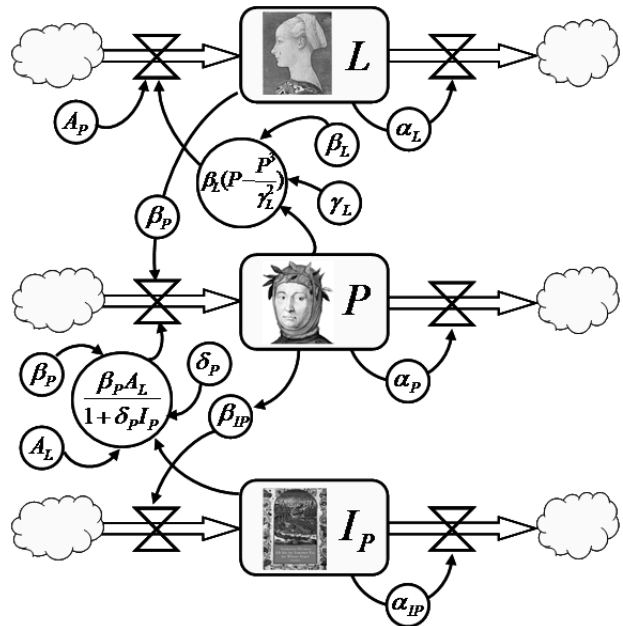


Figure 11: Full stock and flow diagram SD model for Laura's and Petrarch's emotions and for Petrarch's inspiration.

These considerations result first in a table function with few m heuristically derived tabulated values, which directly can be used as auxiliary in SD models:

$$A_{L,I_P}(I_P) = TAB \left((A_{L,1}, I_{P,1}), \dots, (A_{L,m}, I_{P,m}); I_P \right)$$

Mathematically this table function can be approximated by a hyperbolic function (displayed in Figure 10 as dashed curve), which alternatively can be used as auxiliary in the SD model, or for analytical investigations:

$$A_{L,I_P}(I_P) = \frac{A_L}{1 + \delta_P \cdot I_P}$$

Full stock and flow diagram model. With table functions or with analytical functions for Laura's reaction on Petrarch's emotion $R_L(P)$ and Laura's appeal for Petrarch A_{L,I_P} now the full SFD model can be compile, as given in Figure 11.

The 'nonlinear' relations for Laura's appeal and reaction are of different quality. Choosing in the nonlinear cubic-like gain for Laura's reaction $R_L(P)$ a big value for the parameter γ_L , the nonlinear auxiliary becomes almost linear (the nominator is bounded, usually less than 1). The nonlinear auxiliary for Laura's appeal A_L becomes linear, if the parameter δ_P is set to zero, letting the influence of Petrarch's poetic inspiration vanish.

The SFD model allows to derive the ODE model, which in this case is given by

$$\begin{aligned}\frac{dL}{dt} &= -\alpha_L \cdot L + \beta_L \cdot TAB((P_{i1}, R_{L,i}); P) + \beta_L \cdot A_P \\ \frac{dP}{dt} &= -\alpha_P \cdot P + \beta_P L + \beta_P \cdot TAB((A_{L,i}, I_{P,i}); I_P) \\ \frac{dI_P}{dt} &= -\alpha_{IP} \cdot I + \beta_{IP} P \quad \text{or by} \\ \frac{dL}{dt} &= -\alpha_L \cdot L + \beta_L \cdot P \cdot \left(1 - \left(\frac{P}{\gamma_L}\right)^2\right) + \beta_L \cdot A_P \\ \frac{dP}{dt} &= -\alpha_P \cdot P + \beta_P L + \beta_P \cdot \frac{A_L}{1 + \delta_P \cdot I_P} \\ \frac{dI_P}{dt} &= -\alpha_{IP} \cdot I + \beta_{IP} P\end{aligned}$$

This ODE system can be generated automatically by an appropriate SD simulation system, sometimes it is directly translated into a discrete model using Euler integration formula.

4 Identification of Laura–Petrarch SD Model

The big challenge is to identify the model parameters in the nonlinear *Laura-Petrarch Model*, with two appeal parameters, with three gains, with three time constants, and with two parameters for the nonlinearity – in sum ten parameters. A brute-force identification starting with arbitrary values for these parameters is not successful, especially as the appeals may also be negative.

Consequently first the size of the parameters and relations between them should be qualitatively analysed, following S. Rinaldi ([2]). The time constants α_L , α_P , and α_{IP} describe the forgetting processes. For Laura and Petrarch obviously $\alpha_L > \alpha_P$ holds, because Laura never appears to be strongly involved, while the poet definitely has a tenacious attachment, documented by poems:

- In sonnet XXXV Petrarch claims:
*Solo et pensoso i piu deserti campi
vo mesurando i passi tardi e lenti,
.....
Ma pur si aspre vie ne' si selvage
cercar non so ch' Amor non venga sempre
ragionando con meco, et io col' lui.
[Alone and lost in thought, each lonely strand
I measure out with slow and laggard step,
.....
Yet I cannot find such harsh and savage trails
where love does not pursue me as I go,
with me communing, as with him do I.]*

The inspiration of the poet wanes very slowly, be-

cause Petrarch continues to write (over one hundred poems) for more than ten years after the death of Laura. The main theme of these lyrics is not his passion for Laura, which has long since faded, but the memory for her and the invocation of death:

- In Sonnet CCLXVIII, written about two years after Laura's demise, Petrarch remembers:
*Tempo e ben di morire,
et o tardato piu ch'i non vorrei.
Madonna e morta, et a
seco il mio core;
e volendol seguire,
interromper conven quest'anni rei,
perche mai veder lei
di qua non spero, et l'aspettar
m'e noia.
[It's time indeed to die,
and I have lingered more than I desire.
My lady's dead, and with her my heart lies;
and, keen with her to fly,
I now would from this wicked world retire,
since I can no more aspire
on earth to see her, and delay will me destroy.]*

Consequently between the time constants α_{IP} and α_P the relation $\alpha_{IP} < \alpha_P$ must hold. As Petrarch's inspiration holds about ten years, whereas Laura forgets Petrarch in about four months, and Petrarch's passion fades in one year, suitable relations and values are

$$\alpha_L \sim 3 \cdot \alpha_P, \alpha_P \sim 10 \cdot \alpha_{IP}, \alpha_P \sim 1$$

The gains or reaction parameters β_L , β_P , and β_{IP} also can be estimated qualitatively, with respect to the time constants:

$$\beta_L \sim \alpha_P, \beta_P \sim 5 \cdot \alpha_P, \beta_{IP} \sim 10 \cdot \alpha_P$$

Here the assumption is that Laura's reaction equals the forgetting time of Petrarch, and Petrarch reacts five times stronger. For simplicity, the parameters γ_L and δ_P are normalised to one, since it is always possible to scale $P(t)$ and $I_P(t)$ suitably.

The choice of the appeal parameters A_L and A_P is crucial, because these parameters determine the qualitative behaviour of emotion dynamics – cyclic nonlinear behaviour, or damped oscillation toward an equilibrium. In case of Laura and Petrarch, cyclic love dynamics are expected in order to meet the experimentally founded emotional cycle $E(t)$ of Petrarch.

Clearly, Petrarch loves Laura, so for the basic appeal $A_L > 0$ must hold and indeed Laura is a beautiful woman – Figure 12 shows some historic portraits.



Figure 12: Portraits of Laura and Petrarch, from Internet resources.

It is to be noted, that the Laura’s basic appeal A_L is modified by the before given hyperbolic function to Laura’s inspiration-dependent appeal $A_{L,I_P}(I_P)$.

By contrast, Petrarch is a ‘cold scholar interested in history and letters’. He is appointed a *cappellanus continuus commensalis* by Cardinal Giovanni Colonna, and this ecclesiastic appointment brings him frequently to Avignon, where Laura lives. Consequently Petrarch’s appeal A_P is assumed to be negative. Appropriate choices for the appeals A_L and A_P are:

$$A_L \sim 2, \quad A_P \sim -1$$

The negativity of the appeal of Petrarch (see also the portraits in Figure 12) is somehow recognized by the poet himself:

- In sonnet XLV, while Petrarch is talking about Laura's mirror, he says
*Il mio adversario in cui veder solete
 gli occhi vostri ch'Amore
 e'l ciel honora, ...*
 [My rival in whose depths
 you're wont to see
 your own dear eyes which
 Love and heaven apprize, ...]

The above estimated ten parameter values, together with zero initial values for emotion dynamics and for the poetic inspiration, are a good choice for identification. For identification, a least squares method can be used, which minimizes the difference between the data (the grades given by the emotional cycle E_k) and Petrarch’s emotion $P(t_k)$ from the SD model at defined time instants t_k , using the parameter relation derived before:

$$\sum (P(t_k) - E_k)^2 \rightarrow \min$$

Figure 9 shows an identification result for Petrarch’s emotions $P(t)$ vs. Petrarch’s emotional cycle with data $E(t_k)$ (‘graded’ poems).

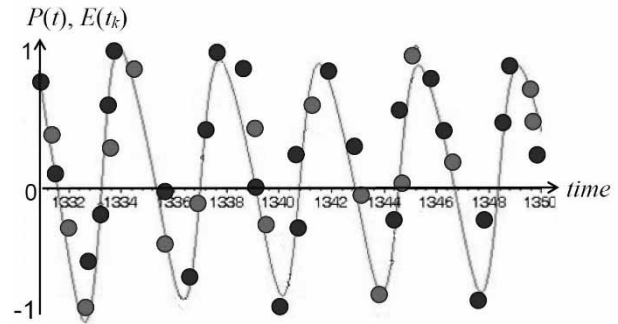


Figure 13: Result of model identification: love dynamics $P(t)$ for Petrarch coinciding with data from Petrarch’s emotional cycle $E(t_k)$, with data E_k (blue and red dots).

Figure 14 shows all results for the identified parameters, structured within a graphical user interface for experimenting with parameter changes. The results of the numerical solution are qualitatively in full agreement with the *Canzoniere* and with the analysis of Frederic Jones. Petrarch’s emotion $P(t)$ tends toward a regular cycle characterised by alternate positive and negative peaks. Also, Laura’s emotion $L(t)$ and Petrarch’s poetic inspiration $I_P(t)$ tend towards a cyclic pattern.

At the beginning, Petrarch’s inspiration $I_P(t)$ rises much more slowly than his emotion and then remains positive during the entire period. This might explain why Petrarch wrote his first poem more than three years after he has met Laura, but then continues to produce lyrics without any significant interruption.

By contrast, Laura’s emotion is always negative. This is in perfect agreement with the *Canzoniere*, where Laura is repeatedly described as adverse:

- In sonnet XXI, Petrarch calls Laura *dolce mia guerrera* [my sweet enemy].
- But in sonnet XLIV Petrarch says:
*ne lagrima pero discese anchora
 da' be' vostr'occhi, ma disdegno et ira.*
 [and still no tears your lovely eyes assail,
 nothing as yet, but anger and disdain.]

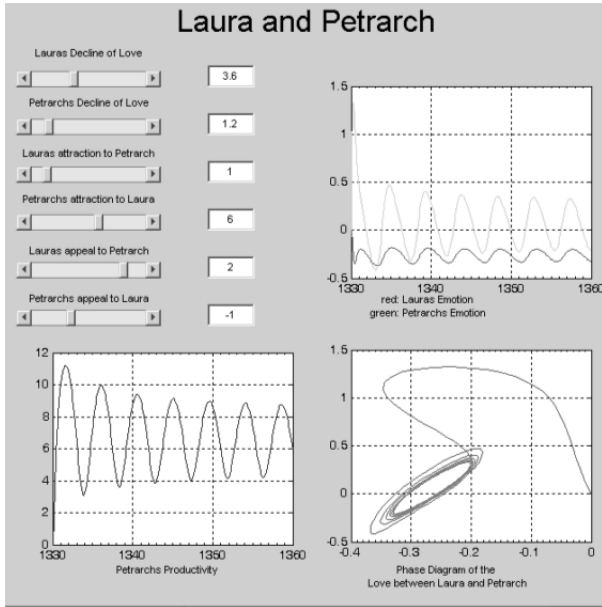


Figure 14: GUI for experimenting with the Laura-Petrarch Model, with parameter values from identification:
 i) above left: sliders for gain β_L , time constant α_L , gain β_P , time constant α_P , and appeals A_L and A_P ;
 ii) upper right: Laura's emotion L (green) and Petrarch's emotion P (red) over time period 1130 - 1360;
 iii) lower left: Petrarch's inspiration I_P , (blue) over time -
 iv) lower right: phase portrait $P(L)$ of love dynamics of Petrarch and Laura – P over L with nonlinear cycle.

5 Experiments with Laura-Petrarch SD Model

Experiments with the parameters show, that the cyclic emotional dynamics may change to a damped oscillation converging to equilibrium. It is difficult to find out which parameter quality causes a cyclic behaviour, and which the damped oscillations. Rinaldi ([2]) has investigated the behaviour carefully, finding a Hopf bifurcation which switches from cyclic to converging stable behaviour. Here two results from simulations with parameter change are shown

5.1 Laura's fast fading emotions

Starting with the classic Laura-Petrarch parameters, an increase of only one parameter α_L by a factor of 2.5 changes the qualitative behaviour essentially (Figure 15) – this parameter change means, that Laura forgets Petrarch in about half time than before. Result is a strongly damped behaviour converging to equilibrium with very small positive and negatives value for P and L , resp.

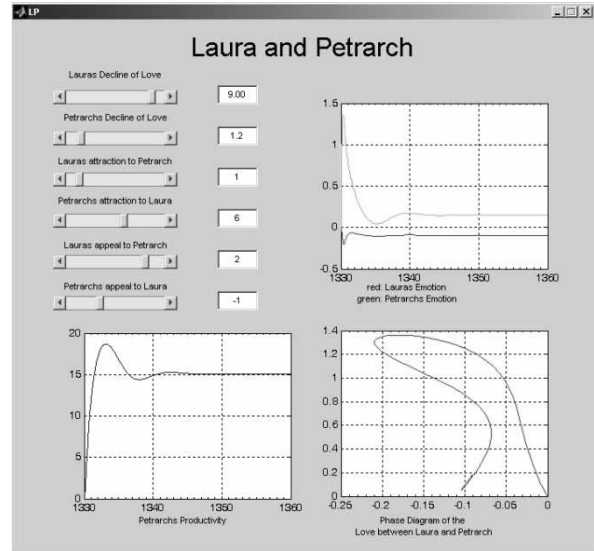


Figure 15: Experiment with Laura-Petrarch Model: Laura's fast fading emotions.

5.2 Petrarch with positive appeal

An interesting experiment is the case of an attractive Petrarch. Supposing e.g. that Petrarch is a young beautiful men, almost like Apollo, he may have the appeal $A_L \sim 6$ to Laura, three times the appeal of Laura to him (all other parameters unchanged).

Figure 16 shows the results: emotions and inspiration are very strongly damped and converge to steady states with relative high positive values – but this is a boring development. This surprising results may conclude, that for non-boring emotions it is necessary, that appeal is opposite.

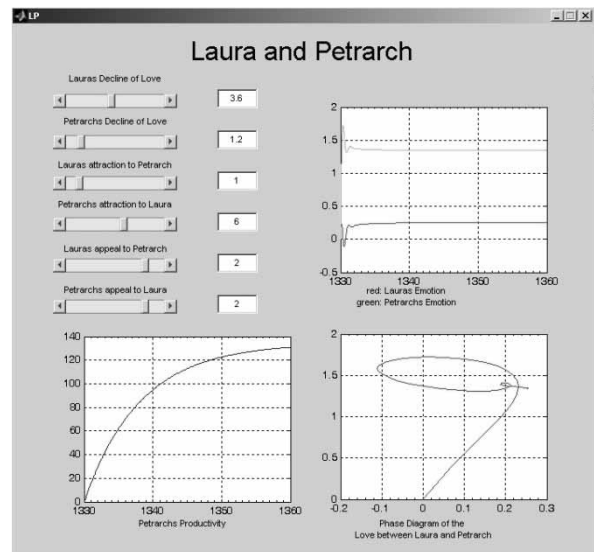


Figure 16: Experiment with Laura-Petrarch Model: Petrarch with positive appeal.

6 Nowadays Couple SD Model

In times of gender equality women as well as men may play an active part in a love affair. Consequently also women express their love by poems or other media, and they confess their love to public. By this, an additional stock with flow for the woman’s inspiration can be introduced easily. For Laura and Petrarch this would mean, that also Laura writes poems, that Petrarch’s appeal is influenced by Laura’s poetic inspiration, and that Petrarch shows more sensibility in his reaction to Laura. Consequently, the structure of the System Dynamics model (Figure 17) suggests a genuine and natural extension: symmetric stocks, flows, and feedbacks gains as well for ‘Petrarch’ and for ‘Laura’, which should now generally represent a man and a woman who show emotions to each other.

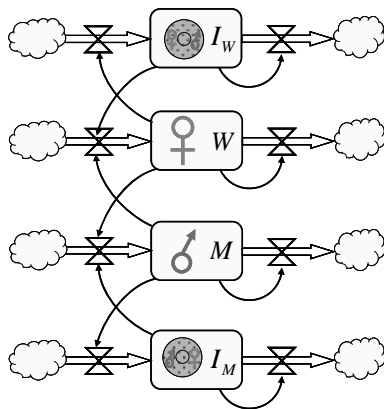


Figure 17: Qualitative SFD model for emotions and inspirations in Nowadays Couple Model.

6.1 Full Nowadays-Couple SD model

The *Nowadays Couple* SD model describes the emotion dynamics $W(t)$ for a woman, and $M(t)$ for a man both falling in love to each other; love inspires both the communicate their love to public, in letters, in videos, with CDs and DVDs, etc. – represented by the inspiration variables $I_W(t)$ and $I_M(t)$.

Also men are now following the more sensitive but more complex behaviour in the reactions to the partner’s emotions. Now, because of the symmetry in emotions and inspirations, the model makes use of two nonlinear cubic-like reaction functions for woman’s and man’s reaction to each other, and of two nonlinear relations between inspiration, appeal, and emotion.

Figure 16 presents the complete nonlinear *Nowadays Couple* SD model in SFD notation,

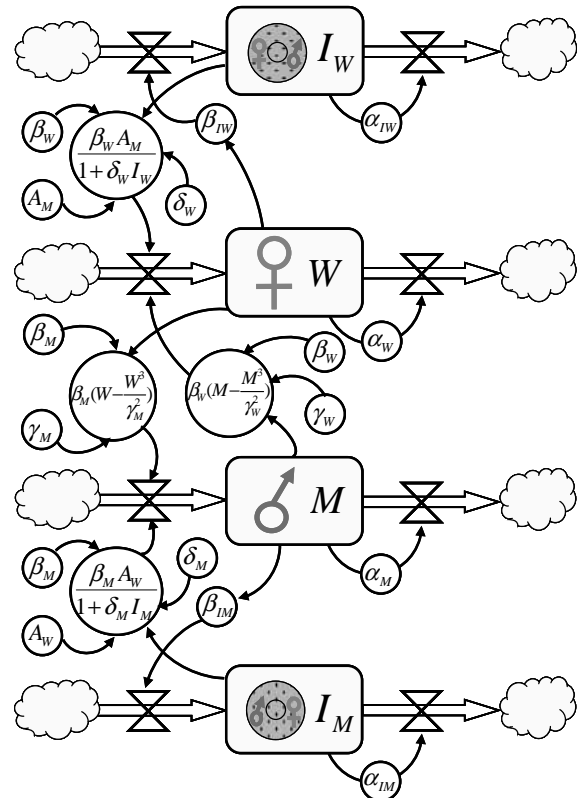


Figure 18: SD Stock and flow diagram for *Nowadays Couple* Model with nonlinear reactions and appeal.

Compared with the *Laura-Petrarch Model*, the *Nowadays-Couple Model* must make use of an increased number of parameters: four fading parameters (instead of three), four (linear) weighting factors for the cross-feedbacks (instead of three), two appeal parameters (instead of one), and four parameters in the nonlinear functions (instead of two) – in sum 14 parameters.

A theoretical analysis of this model is almost impossible, but numerical experiments may give interesting insight into emotion dynamics. The ODE model derived from the SFD (Figure 18) shows – as the SFD – a symmetric structure:

$$\frac{dW(t)}{dt} = -\alpha_W W(t) + \beta_W M \left(1 - \left(\frac{M}{\gamma_W} \right)^2 \right) + \beta_W \frac{A_M}{1 + \delta_W I_W(t)}$$

$$\frac{dI_W(t)}{dt} = -\alpha_{IW} I_W(t) + \beta_{IW} W(t)$$

$$\frac{dM(t)}{dt} = -\alpha_M M(t) + \beta_M W \left(1 - \left(\frac{W}{\gamma_M} \right)^2 \right) + \beta_M \frac{A_W}{1 + \delta_M I_M(t)}$$

$$\frac{dI_M(t)}{dt} = -\alpha_{IM} I_M(t) + \beta_{IM} M(t)$$

6.2 Experiments with Nowadays-Couple SD model

As with the *Laura-Petrarch Model*, the *Nowadays Couple Model* has been implemented in a graphical user interface (GUI) for experimenting with parameters and displaying solutions: emotion dynamics with cycle limit, convergence to stable constant emotion limit (with few or many waves, with positive and / or negative limit value for emotion), etc. Among 14 parameters, it is difficult to find parameters for specific behaviour.

For demonstration purposes, therefore a simplified GUI has been developed, which allows selection of certain specific cases with predefined parameters. Figure 19 and Figure 20 present two of these case studies:

- ‘Everyday Boring’: Almost no waves in the emotions, fast convergence to a stable constant emotion value (positive for women, negative for man)
- ‘Pretty and Ugly’: Opposite parameters (one pretty, one ugly) result in a fast waves in both emotions for the first five years, then convergence to a relatively high constant emotion value for both.

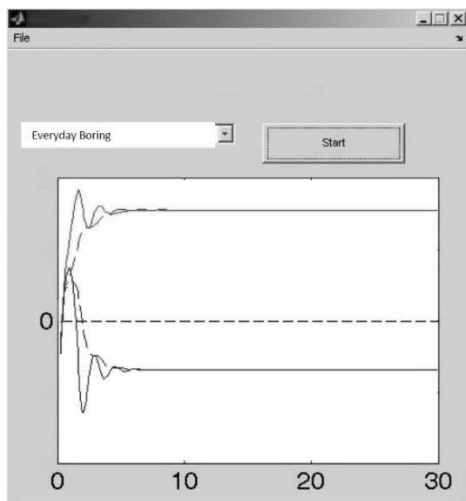


Figure 19: Nowadays Couple Model
Experiment ‘Everyday Boring’
- red/blue – woman’s/men’s emotion;
- red/blue dashed – woman’s/men’s inspiration).

Time-dependent appeals. Does the *Nowadays Couple Model* reflect reality? The model is able to mimicry different situations, but with one assumption: the general appeal parameters A_M and A_W are constant up to now (note: they are multiplied by the hyperbolic functions depending on inspiration, but themselves they are constant). This assumption may not meet reality; the appeal for each other may change over time, e.g. aging, and they also may be manipulated and controlled.

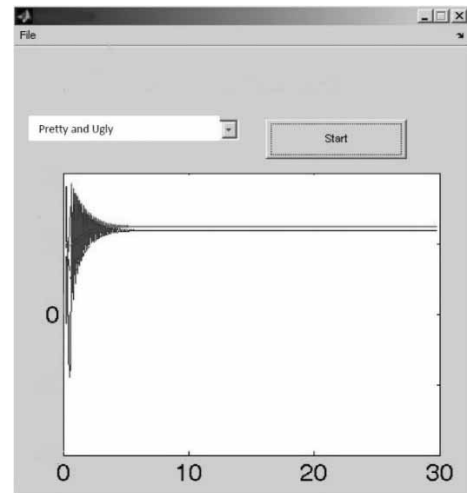


Figure 20: Nowadays Couple Model
Experiment ‘Pretty and Ugly’
- red/blue – woman’s/men’s emotion.

A dynamic appeal can be easily modelled by time-dependent general appeal variables $A_M(t)$ and $A_W(t)$, either by a specific mathematical time function, or simply by an only time-dependent table function:

$$A_M(t) = TAB \left((A_{M,1}, t_1), \dots, (A_{M,n}, t_n); t \right)$$

Case studies may become now very complicated, because not only 14 parameters have to be chosen appropriately, but also the function $A_M(t)$ and $A_W(t)$ have to be provided meaningful.

An extended version of the GUI presented in the two figures before allows additionally providing predefined appeal functions. Figure 21 and Figure 22 show results for perhaps interesting cases:

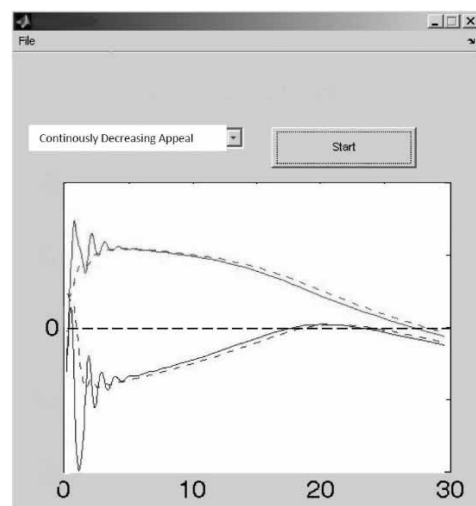


Figure 21: Nowadays Couple Model
Experiment ‘Continuously Decreasing Appeal’
- red/blue – woman’s/man’s love emotion;
- red/blue dashed – woman’s/man’s inspiration).

- ‘Continuously Decreasing Appeal’: the appeals decreases exponentially, resulting in a convergence of the emotions to small values
- ‘Jump in Appeal – Aesthetic Surgery’: after ten years of fading emotions, e. g. an aesthetic surgery increases the women’s appeal, resulting in a jump of emotions – into positive for her, into negative for him, but followed by same emotion fading than before.

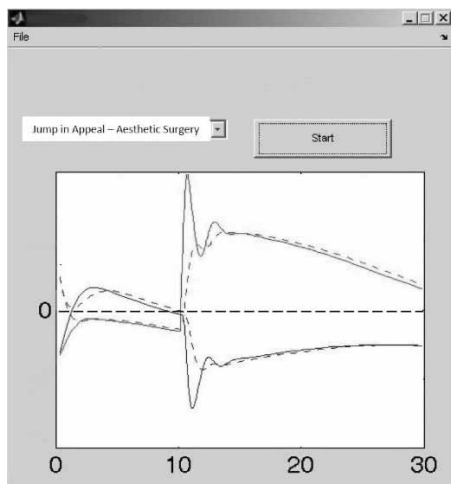


Figure 22: Nowadays Couple Model
Experiment ‘Jump in Appeal – Aesthetic Surgery’
- red/blue – woman’s/man’s love emotion;
- red/blue dashed – woman’s/man’s inspiration.

7 Conclusion

The method of *System Dynamics* is indeed a valuable tool for modelling social systems – as the investigated system of emotions. In contrary to engineering, no precise forecasts can be made, only scenarios can help for better understanding.

Of course, this contribution presents serious investigations. But is it possible to investigate the dynamics of emotions, perhaps the most important phenomenon concerning our lives, seriously by methods of mathematics and engineering? One could also conclude, it might be better not to tackle the secrets of love, because described and controlled by formula, it is not love anymore longer. In this view, the contribution might be seen as reference to Petrarch and the most beautiful love poems the author ever read.

Modelling methodology provides a classification for models – where also the type *Verbal Model* can be found. On occasion of Petrarch’s 800th birthday, his sarcophagus was opened, and near to Petrarch’s skull a bottle was found (Figure 23), with a manuscript of a sonnet:



Figure 23: Bottle with manuscript of sonnet, in Petrarch’s sarcophagus.

- *Benedette le voci tante ch’io chiamando il nome de mia donna ò spare, e I suspire, et le lagrime, e ’l desio; et benedette sian tutte le carte ov’io fama l’acquisto, e ’l pensier mio, ch’è sol di lei, sí ch’altra non v’à parte.*
- [*And blessed be all of the poetry I scattered, calling out my lady’s name, and all the sighs, and tears, and the desire; blessed be all the paper upon which I earn her fame, and every thought of mine, only of her, and shared with no one else.]*

Perhaps this sonnet is the best model for the emotions expressed in poems – the *Verbal Model* for the emotions is the sonnet itself.

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