Anatomical Joint Constraint Modelling with Rigid Map Neural Networks

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Abstract. The development of anatomical models both for individuals and groups are important for applications in animation, medicine and ergonomics. Recent approaches have utilised unit quaternions to represent orientations between limbs which eliminate singularities encountered in other rotational representations. As a result a number of unit quaternion based joint constraint validation and correction methods have been developed. Recent approaches harness machine learning techniques to model valid orientation spaces and has included the use of Kohonen's Self Organizing Maps (SOMs) to model regular conical constraints on the orientation of the limb. Recent work has considered a derivative of the SOM, the Rigid Map, applied in the same context which we extend here.

Introduction

Anatomically correct joint models are essential to ensure realistic movement during simulation for applications in animation, medicine and ergonomics [1, 2, 3]. Many current approaches are limited by their underlying representation of rotation or abstraction of the joint function [4], while in others, accuracy is linked to computational cost [5]. This work builds on previous research exploring the use of machine learning to model joint constraints; specifically using unsupervised neural networks to model unit quaternion based phenomenological [6] joints (whose behaviour can be modelled without reference to the underlying joint anatomy).

Rigid Maps [7], similar to Kohonen’s Self Organising Map (SOM), are used to implicitly model the boundary between valid and invalid orientations by modelling a group of valid rotations, expressed as unit quaternions. The SOM produces a topography preserving projection of the prototypes from the n-dimensional input space onto an m-dimensional output space [8], while the Rigid Map [7] uses a fixed output space of uniformly distributed unit quaternions. Competitive learning is employed to train a Rigid Map to represent a group of valid unit quaternion orientations. In response to an input orientation, the output is the weight of the output node which best matches the input, this can be used to provide a target for correction.

This paper considers constraints on the rotation of the limb (or swing [9]) with regular (circular) bounded constrained regions. Irregular boundaries and rotation around the limb (or twist [9]) are the subject of future work.

1 Background

Joint constraints can be expressed using Euler angles: this box-limit model is popular in animation tools and file formats [4]. Such course representations fail to capture inter-dimensional dependencies [10] and can encounter singularities [11]. Inter-dimensional dependencies can be represented by geometric functions fitted to a given data set e.g. spherical [12] and conical polygons [1]. Alternative rotational parameterisations have been deployed to overcome singularities including special orthogonal matrices [2] and unit quaternion e.g [13].

Quaternions are an extension of complex numbers, a subgroup where all quaternions are of unit length (the unit quaternion group) and their associated algebra allows the representation of rotation without the presence of gimbal lock [11]. Unit quaternions occupy a three dimensional surface (a hypersphere) in four dimensional space. This mapping is redundant as the unit quaternion represent 4π rotations, polar opposites (q and −q) describe the same orientation [11].

Unit quaternion joint constraints can be modelled by decomposing the limb origination, as a unit quaternion, into conical and axial components (also unit
Artificial Neural Networks (ANNs) have been employed to model anatomical joint constraints represented using unit quaternions. Here, unlike other approaches [13, 15], unit quaternions can be used as input without decomposition or projection. ANNs have been trained using supervised learning approaches to implicitly model a joint constraint boundary [18]. Such approaches are difficult to apply to recorded data as they require both valid and invalid patterns for training. To overcome this issue, ANNs trained using unsupervised techniques such as competitive learning have been proposed. SOMs have been trained using competitive learning to implicitly model joint constraints using only valid orientations expressed as unit quaternions [19, 20]. The weights of the output nodes are trained via competitive learning to represent the training data while preserving the topography of the input space. The network responds to a given input orientation with the closest orientation in its model of the input data. This can be used directly for correction [19] or as a target for an iterative approach [20].

The Rigid Map Network is a modified SOM proposed for pose estimation problems by Winkler et al [7]. In their approach self-organisation is abandoned, the output node topology is fixed and the nodes are uniformly distributed over the orientation space, in this case the $S^3$ hypersphere using regular polyhedra. The learning algorithm is modified such that during training the winning node is based on the proximity between the input pattern and the position of the output node, rather than its weight (as in the SOM), determined by the inner product. The updating of weights, however, remains unchanged with the weight of the winning node and its neighbours being moved some distance toward the input according to the learning rate [21]. Both the learning rate and the radius of the neighbourhood decay exponentially with time [21]. When fired, the network responds with the weight which is the shortest Euclidean distance from the input [21].

It is hypothesised that the Rigid Map Network will produce superior results to the earlier SOM approach as the orientation space is known and self-organisation can be abandoned. Exploratory work has considered the capabilities of the Rigid Map in modelling the orientation of the limb with a regular rotational boundary and no constraint on the rotation around the limb. Future work will explore more complex constraints including irregular boundaries and rotation around the limb.

The remainder of this paper is structured as follows: Section 3 provides a description of our methodology with reference to the techniques employed. Section 4 reports the results of the experiments undertaken with these discussed in Section 5. Finally Section 6 draws conclusions from this work and highlights areas for future investigation.

## 2 Methodology

The Rigid Map used consists of four input nodes and a number of output nodes joined by a weighted connection. The output nodes are placed into a topology each having a position on the unit quaternion hypersphere, arranged using a selection of regular polytopes in 4D-space, in this case the polydodecahedron and polytetrahedron. The polytetrahedron has 120 vertices and 600 tetrahedral cells, while its reciprocal, the polydodecahedron has 600 vertices and 120 dodecahedral cells [22]. Combining these results in the vertices of the polydodecahedron being placed at the center of the polytetrahedron [21].

The Rigid Map was trained according to the process defined by Winkler et al [21, 7]. Each experiment was repeated ten times to ensure the consistency of the results. The Rigid Map used in this work was based on that presented by Winkler [21] modified such that the output nodes occupy the whole hypersphere rather than a single hemisphere.

Experiments were undertaken with output nodes arranged as polytetrahedron, polydodecahedron and a combination of both with on datasets of between 500 and 6000 patterns. In experiments where the range was
not varied, a constant range of $90^\circ$ was used with other training parameters identified through experimentation. The training dataset contained only valid patterns, similar to those recorded from the movement of a human arm. A set of ‘ideal’ corrections (no correction for valid orientations and the nearest valid orientation for invalid,) were generated using Lee’s [13] approach and provided a measurement of the Rigid Maps capabilities.

3 Results

The results show the effect of correcting the orientation to that suggested by the Rigid Map (the unit quaternion represented by the weight of the winning node), indicating successful training of the Rigid Map. An increase in the range (angle between the virtual limb and the z-axis) of the constrained region results in a decrease in performance, as shown in Figure 1. The resulting corrections, however, are inferior to those of the SOM (from our earlier work [19]) using the same training data, training iterations and a similar number of output nodes (625) as shown in Figure 1.

![Figure 1: Performance of the Rigid Map with increasing constraint range compared to a similar SOM.](image)

Increasing the number of training epochs produced an increase in performance, which attenuates rapidly as the number of epochs increases. The network error decreases as the number of output nodes is increased while the error appears independent of the number of training patterns.

4 Discussion

The results demonstrate that Rigid Maps are capable of identifying the nearest unit quaternion representing a valid orientation of a virtual anatomical limb, providing a representation of a region occupied by valid orientations in unit quaternions space. The Mean Squared Error (MSE) on the test set (containing invalid and valid orientations) is reasonably low, but higher than those for the SOM (shown in Figure 1). As in earlier work overcorrection is a problem; the limb is corrected to the orientation provided by the weights of the winning node, these being inside the valid region, while the testing process measures the MSE based on the distance from the boundary.

The results provide an insight into the effects of problem, network and training attributes on performance. It is clear that the network is capable of learning constraints of varying sizes, although larger constraints appear to demonstrate a higher error. This suggests an increase in overcorrection of valid points as output nodes are more dispersed over the valid region and an increase in overcorrection of invalid points as fewer output nodes occupy spaces near the boundary. Improvements resulting from the increase in output nodes can be ascribed to an increase in the density of output nodes over the valid region, reducing correction errors. Winkler [21], recommends an even distribution of output nodes, however no further polytopes exist [22]. This has implications for both small networks and large constraints due to the low density of output nodes in the valid region.

Previous results with the SOM [19] network showing improved results with an increase in the data set size are not echoed in the results for the Rigid Map, suggesting that the other factors (possibly the limited output node density) limit further improvements in performance.

5 Conclusion

Rigid Maps have been shown to be capable of representing a group of valid orientations in unit quaternion space to a degree of accuracy. However, this requires that the output nodes are uniformly distributed in the output space [21]. This initial research shows them to demonstrate inferior performance to the traditional SOM. Both approaches have similarities to non-machine learning based solutions [16, 17] with the advantage that no decomposition or reformatting of the unit quaternion orientation is required. Comparisons with other popular approaches in terms of accuracy and speed are now required.

Research is required into the tuning of the Rigid
Maps training parameters, along with the distribution and density of nodes in the output layer. A key limitation of this work, highlighted by the results, is an inability to explore larger output layers. Subdivision of the regular polytopes [7] along with other techniques [23] are being explored as part of our ongoing research.

Current results are encouraging and suggest that Rigid Maps are able to implicitly model constraints on the rotation of the limb with regular boundaries in unit quaternion space. They may have potential for modelling similar constraints with irregular boundaries and rotation around the limb while providing advantages over current approaches.

References


