

# Modeling, Simulation, and Optimization with Petri Nets as Disjunctive Constraints for Decision-Making Support. An Overview.

Juan-Ignacio Latorre-Biel<sup>1\*</sup>, Emilio Jiménez-Macías<sup>2</sup>

<sup>1</sup>Department of Mechanical, Energetic, and Materials Engineering, Public University of Navarre, Campus of Tudela, 31500 Tudela, Spain; \*[juanignacio.latorre@unavarra.es](mailto:juanignacio.latorre@unavarra.es)

<sup>2</sup>Department of Electrical Engineering, University of La Rioja, 26006 Logrono, Spain

Simulation Notes Europe SNE 26(2), 2016, 75 - 82  
 DOI: 10.11128/sne.26.on.10333  
 Received: June 18, 2016 (Invited Overview Note);  
 Accepted: June 25, 2016;

**Abstract.** A panoply of modeling formalisms, based on the paradigm of Petri nets is overviewed and their application to modeling, simulation, and optimization of discrete event systems with alternative structural configurations is discussed. This approach may be appropriate for the development of decision support systems for the design process of discrete event systems. The motivation, definition and an example of application is provided for several formalisms that include a set of exclusive entities. A practical methodology and the main advantages and drawbacks of the application of these formalisms to the calculation of quasi-optimal values for the freedom degrees in the structure of discrete event systems in process of being designed is addressed.

## Introduction

The application of discrete event systems (DES) to a broad variety of fields of technological interest is growing from day to day [1], [2].

Petri nets (PN) consist of a paradigm widely used for the modelling, simulation, and optimization of DES. Many theoretical results related to the PN contribute to the body of knowledge that can be applied to the construction of models, their simplification and verification by structural analysis, their validation, as well as the implementation of performance analysis [3], [4].

Simulation consists of another very productive methodology for many operations related to PN. In particular diverse decision-making support methodologies rely upon the simulation of the evolution of the Petri net model of an original DES [5], [6], [7].

Several methodologies for decision-making support based on the simulation of Petri net models proceed by means of the following steps:

- a) Obtaining a Petri net model of a DES with freedom degrees or controllable parameters.
- b) Defining the objectives to be achieved by the DES and the way to evaluate their degree of achievement, also called quality, in its Petri net model.
- c) Making a guess on a feasible (and promising) set of values for the different freedom degrees of the model, or solution of the decision-making problem, including the initial state or marking of the Petri net.
- d) Configuring the parameters of the simulation, for example the stop criterion. In some cases, such as for a manufacturing process, this criterion might be the completion of a certain period of simulated time.
- e) Developing the evolution of the Petri net, while gathering the information necessary for evaluating the quality of the tested solution.

These methodologies reach a significant potential if the previously mentioned steps from (a) to (e) are iterated for different solutions or values for the controllable parameters of the model. After a number of solutions have been simulated, regarding the availability of time and computer resources, it is possible to provide a set of selected solutions to the decision maker [8], [6], [9], [10].

Depending on the way of choosing the feasible solutions to be simulated, several methodologies can be defined, since an exhaustive exploration of the complete solution space is not possible for the majority of cases. This limitation arises from the construction process of the set of feasible solutions for the freedom degrees of the Petri net. This may be carried out by a combinatorial process, where different values for the diverse controllable parameters are combined for building up solutions.

As a consequence, the size of the solution space might be huge and, since the simulation of a single solution may consume significant computer resources, the simulation of all the solutions might not be a realistic option [6], [9], [10].

In particular, a linear augmentation of the size of the set of controllable parameters implies an exponential growth of the size of the solution space.

Once the exhaustive search for solutions is discarded, a guided search of the most promising solutions is a practical approach. Nevertheless, due to the fact that the exploration is reduced to a small region of the solution space, the optimum is not always found. However, a good solution, quasi-optimal, is enough for practical purposes.

This search for promising solutions can be a manual process, such as a “what-if” analysis, where small changes on tested solutions seek to deduce the influence of variations of one or several freedom degrees’ values in the outcome of the simulation [11], [12]. Alternatively, an automatic procedure for exploring feasible solutions may use of metaheuristics for exploring local optima in the search for the global optimum [5] to [10].

This approach has been applied to the operation of a DES [5] to [8], [10]. That means that the makespan, yield, utilisation rate of the equipment, or level of stocks have been considered to evaluate the quality of solutions for the scheduling or routing of systems such as a manufacturing facility or a supply chain, just to give an example.

Another common and complex case, where decision making support can be applied is the design of a DES [13], [9]. One of the key steps in the design of a system is the analysis of different alternative solutions for choosing the best one and proceed with the following design stages. It is very common that different alternative solutions for the design of a DES can be modelled by PN with different static structures [13], [9].

The design of a DES is not the only application dealing with a variety of static structures. It is the case of systems, whose structure varies over time on a controllable process, i.e. a decision maker should find the best sequence of transformations to achieve its goals. An example is the problem of preventive maintenance, where a manufacturing facility should stop sequentially its different installations, without a restriction on the sequence to follow but with strong constraints on the period of time each installation is stopped, the resources used in the operations of maintenance, the overall loss of production and the impact on the service provided to customers [14].

The way to build up the alternative structural solutions to these decision problems might be based in the combination of feasible subsystems of the model [15]. This is especially true, when the DES to be designed is composed by differentiated and real subsystems, such as machines or production lines. The number of feasible alternative solutions arisen from this combinatorial process may be very large; thus, manual selection of the best alternative solution by expensive experts in manufacturing management is not always an efficient perspective.

A decision problem can be stated with a set of alternative PN modelling the structural configurations, which are exclusive constraints; hence, they constitute a disjunctive constraint to the decision problem [16].

Four approaches for solving the problem are addressed in this document, as well as their advantages and drawbacks. Main difference between these approaches is the Petri net formalism chosen for modelling the DES. In brief, the formalism drives the methodology applied to state and solve the decision problem [17], [9].

In Section 1, the formalism of the alternative PN is discussed, while in Section 2, it is considered the compound PN. The following two sections address the alternatives aggregation Petri nets (AAPN) and the disjunctive coloured Petri nets, respectively. Section 5 focusses on the comparative of the methodologies arisen from the different formalisms, while the following section is devoted to the combination of different formalisms in a single PN model. Section 7 describes the common steps of the methodologies for solving a decision problem using these formalisms. Section 8 presents the conclusions and the last section lists the bibliographical references.

## 1 Alternative Petri nets

This formalism has been used in diverse applications and can be considered a classic approach for solving decision problems with PN as disjunctive constraints. A definition and contextualisation is provided in [18].

### 1.1 Motivation

This formalism is a classic approach, used in diverse applications, such as [11], [19], and [12]. It is a natural, yet inefficient way to represent a disjunctive constraint in terms of PN [9]. This intuitive way to describe a set of alternative models for designing a DES consists of developing independent models for every alternative structure.

This simple idea is behind a set of alternative Petri nets, which contains so many nets as alternative models have been considered in the process.

## 1.2 Definition

A set of alternative Petri nets is a collection of models with different structure, i.e. incidence matrix, where any of them is able to describe the same DES. These models can be represented by any Petri net formalism.

**Definition 1. Set of alternative Petri nets.**

$S_R = \{ R_1, \dots, R_n \}$ , set of PN, is a set of alternative Petri nets if

i)  $\text{card}(S_R) = n > 1$ .

ii)  $\forall i, j \in \mathbf{N}^*$  such that  $1 \leq i, j \leq n, i \neq j, R_i, R_j \in S_R$ , then  $\mathbf{W}(R_i) \neq \mathbf{W}(R_j)$ , incidence matrices of  $R_i$  and  $R_j$ .

iii)  $\exists! R_k \in S_R$ , such that  $\mathbf{m}_0(R_k) \neq [0 \ 0 \ \dots \ 0]^T$ , where  $\mathbf{m}_0(R_k)$  is the initial state or initial marking of  $R_k$ .  $\square$

In other words, given a set of alternative Petri nets associated to a DES, the choice of any of them as solution for the structure of the system's model implies that the initial marking of all the non-chosen alternative Petri nets contains zero tokens in every place. Only the chosen PN can be simulated at a given time. The simulation of any other PN requires discarding the previous choice.

## 1.3 Examples

Some examples of sets of alternative Petri nets are:

A decision problem for deciding the best structural configuration of a manufacturing facility among a set of three alternative Petri nets is stated in [18]. A second example is provided just to illustrate the statement of an optimization problem with such a disjunctive constraint.

Four different topologies of manufacturing facilities, with diverse degrees of production flexibility, are discussed in [12]. Their Petri net models, developed for simulation, constitute a set of four alternative Petri nets.

An assembly line is modelled in [19] under three different manufacturing strategies. The resulting alternative Petri nets are simulated for choosing the best control policy: push, "on demand", or Kanban.

A manufacturing facility is presented in [11], such that the combination of diverse production strategies and lot sizes lead to a set of alternative Petri nets.

## 2 Compound Petri Nets

This formalism is well known under the name of parametric Petri nets or parameterized Petri nets. An application of this formalism to decision problems with PN as disjunctive constraints is described in [20].

## 2.1 Motivation

In design problems, the alternative Petri nets might be an inefficient option to construct a Petri net model appropriate for simulation because of the following reasons:

- Different alternative solutions might share common subsystems; hence, a complete set of alternative Petri nets may contain a large amount of redundant information. The larger the model, the more computational resources might consume its simulation.
- Each alternative PN defines an independent problem of searching good solutions. Hence, computer resources should be devoted to every alternative Petri net, no matter if it leads to good solutions or not.
- To avoid the statement of a large number of search problems, a manual pre-selection of a small set of feasible solutions is usually carried out. This process might skip high quality solutions.

A compound PN tries to remove these limitations by:

- Removing redundant information in the static structure of the PN that defines the disjunctive constraint.
- Allowing a single search process. Non-promising solutions can be skipped, devoting more computer resources to promising regions of the solution space.
- A single search problem is stated; hence, a manual pre-selection of feasible solutions is not necessary.

## 2.2 Definition

The static structure of a PN is described by its incidence matrix; hence, a compound Petri net presents freedom degrees or controllable parameters in its incidence matrix [21]. Moreover, every alternative structural configuration (ASC) of the model is associated to a feasible set of values for these parameters. Even though it might be possible to construct ASC from different combinations of values for these controllable parameters, in general, not all the combinations are valid sets.

**Definition 2. Compound Petri net.**

A marked compound Petri net is a 7-tuple

$R^c = \langle P, T, \text{pre}, \text{post}, \mathbf{m}_0, S_{\text{str}\alpha}, S_{\text{val}\text{str}\alpha} \rangle$ , where

- $P$  and  $T$  are disjoint, finite, non-empty sets of places and transitions respectively.
- $\text{pre}: P \times T \rightarrow \mathbf{N}$  is the pre-incidence function.
- $\text{post}: T \times P \rightarrow \mathbf{N}$  is the post-incidence function.
- $\mathbf{m}_0(R^c)$  is the initial marking of the net.
- $S_{\text{str}\alpha} \neq \emptyset$  is the set of structural parameters of  $R^c$ .
- $S_{\text{val}\text{str}\alpha}$  is the set of different feasible combinations of values for the structural parameters of the net.

### 2.3 Examples

[21] illustrates the main concepts of a compound PN, while two examples in [20], describe a transformation of a set of alternative PN into one compound PN.

## 3 Alternatives Aggregation Petri Nets

AAPN is a formalism proposed as a tool for describing disjunctive constraints in the form of a Petri net model.

### 3.1 Motivation

Similarly to compound PN, AAPN aim at removing the limitations of a set of alternative models for a DES. The construction process and specific parameters of the formalism itself are discussed in [18].

### 3.2 Definition

An AAPN integrates in a single Petri net model a complete set of ASC. A set of choice variables allows choosing one of the ASC. A function of choice variables is associated to some transitions as guards. Only one choice variable can be active at a given time [16].

**Definition 3. Alternatives aggregation Petri net.**

An AAPN system,  $R^A$ , is defined as the 7-tuple:

$R^A = \langle P, T, \text{pre}, \text{post}, \mathbf{m}_0, S_A, f_A \rangle$ , where,

i)  $P, T, \text{pre}$ , and  $\text{post}$  are explained in Definition 2.

ii)  $\mathbf{m}_0$  is the initial marking representing the initial state and is usually a function of the choice variables.

iii)  $S_A = \{a_1, a_2, \dots, a_n \mid \exists! a_i=1, \text{ where } i \in \mathbf{N}^*, 1 \leq i \leq n \wedge \forall j \neq i \text{ then } a_j=0\}$ .  $S_A$  is a set of choice variables, such that  $S_A \neq \emptyset$  and  $|S_A| = n$ .

iv)  $f_A: T \rightarrow \{0, 1\}$  is a function that assigns a function of the choice variables to each transition  $t$  such that  $\text{type}[f_A(t)] = \text{Boolean}$ .  $\square$

### 3.3 Examples

The application of an AAPN to the design of a manufacturing facility is shown in [18]. Different strategies for solving this problem by means of distributed computation are presented and their computation time compared. The transformation algorithm from a set of alternative Petri nets into a single AAPN is illustrated in [16].

## 4 Disjunctive Colored Petri Nets

Coloured Petri nets consists of a formalism, closely related to the AAPN [22], [9].

### 4.1 Motivation

Even though AAPN can be very efficient, when describ-

ing a model of a disjunctive constraint, simulating the evolution of a model might require specific *ad hoc* tools.

Coloured Petri nets are conceptually similar to the AAPN, but the formalism contains the same elements than the commonly used coloured Petri nets. For this reason, tools developed for simulating coloured Petri nets or the more general high-level Petri nets, can be used for simulating disjunctive coloured Petri nets.

### 4.2 Definition

A disjunctive coloured Petri net presents a static structure which is the same as an equivalent AAPN. The main difference is associated to the mechanism used to decide the chosen alternative structural configuration. In the case of a disjunctive coloured Petri net, it is possible to define a choice colour in different ways [22].

Nevertheless, a choice colour allows for the tokens of a certain structural configuration to describe the evolution of the Petri net [9]. An important characteristic of the choice colour is that it should be monochrome, i.e. it is not possible to mix tokens from different choice colours for analysing the evolution of the Petri net by simulation. This limitation prevents the exploration of unreal states for the Petri net model.

**Definition 4. Disjunctive colored Petri net**

A disjunctive colored Petri net  $R = \langle N, \mathbf{m}_0 \rangle$  is a 9-tuple

$CPN = \langle P, T, F, \mathbf{m}_0, \Sigma, V, \text{cs}, g, e \rangle$ , where:

i)  $P$  and  $T$  are disjoint, finite, non-empty sets of places and transitions respectively.

ii)  $F \subseteq P \times T \cup T \times P$  is a set of directed arcs.

iii)  $\mathbf{m}_0$  is the initial (monochrome) marking.

iv)  $\Sigma$  is a finite set of non-empty color sets, such that verifies one of the following two conditions:

a.  $\exists S_C$  set of Boolean choice colors such that  $S_C \in \Sigma$ .

b.  $\exists (c, C)$  a natural choice color such that  $C \in \Sigma$  and  $C$  is the number of ASC, while  $c \in \mathbf{N}$  and  $1 \leq c \leq C$ .

v)  $V$  is a finite set of typed variables such that  $\text{type}[v] \in \Sigma$  for all variables  $v \in V$ .

vi)  $\text{cs} : P \rightarrow \Sigma$  is a color set function that assigns a color set to each place.

vii)  $g : T \rightarrow \text{EXPR}_V$  is a guard function that assigns a guard to each transition  $t$  such that  $\text{type}[g(t)] = \text{Boolean}$ .

viii)  $e : F \rightarrow \text{EXPR}_V$  is an arc expression function that assigns an arc expression to each arc  $a$  such that  $\text{type}[e(a)] = c(p)_{MS}$ , where  $p$  is the place connected to the arc  $a$ .

### 4.3 Examples

A decision problem on the choice of the best production strategy for a manufacturing facility is given in [9]. In this example a set of 24 alternative Petri nets are transformed into a single disjunctive coloured Petri net. An optimization problem is stated and a comparison between a classic solving strategy by means of the set of alternative Petri nets and the use of the single disjunctive coloured Petri net is shown.

In [22] an application of the disjunctive coloured Petri nets to the design of a manufacturing facility is presented. The modelling process from a set of diverse alternative Petri nets is detailed.

## 5 Advantages and Drawbacks

The main purpose of the four Petri net formalisms presented in this paper, consists of minimizing the size of a Petri net model with a set of ASC. Nevertheless, there are other considerations that may be considered as advantages or drawbacks of these formalisms. In particular, the following features are interesting ones:

- a) Size rate, or quotient between the size of the model, measured by the size of its incidence matrix and the size of an equivalent benchmark, usually a set of alternative Petri nets.
- b) Easiness of modelling. Simulation can be considered as an inexpensive methodology for experimentation, when compared with practising the real system. However, constructing models fast and with absence of errors may make feasible a decision making support tool.
- c) Availability of theoretical and practical tools for analysing, simplifying, and simulating a model of a DES.

It is interesting to realize that the compactness of the Petri net model, allowed under certain conditions by some formalisms presented in this document, does not compromise the usefulness of the model to explicitly represent and show the structure of the modelled system. In fact, the removal of redundant data present in the model of the system tends to point out the key information that determine the structure of the system itself.

Some of the features mentioned in the present section depend on the DES to be modelled. In particular, if there are similarities between the ASC it is likely that certain formalisms might lead to reductions in the size of the model. Of course, it is necessary to specify what means similarity in this context.

The concept of similarity depends on the formalism that is intended to be applied.

For example, a compound Petri net may present a small size rate, corresponding to a large amount of removed redundant data, when the incidence matrices of the alternative Petri nets present a reduced number of different elements. As a consequence, in the context of a compound Petri net, similarity is a concept that can be quantified in inverse proportion to the number of different elements between the incidence matrices of and equivalent set of alternative Petri nets.

Nevertheless, similarity in the case of AAPN or disjunctive coloured Petri nets can be quantified as a parameter proportional to the number and size of shared subnets. This parameter also depends in inverse proportion to the elements of the model that do not belong to any shared subnet. It is not unusual in a design process that the different ASC are built up by means of combining in different ways a given set of subnets [17], [15].

It is also possible to say that if the similarity of the alternative Petri nets that model a DES is not high, the size of the set of alternative Petri net may be smaller than the size of an equivalent model described by any other of the formalisms presented in this document.

Regarding other important feature, such as the modelling easiness, it is possible to say that the alternative Petri nets may lead to a very intuitive way of modelling, since an independent model is developed for any of the ASC. Nevertheless, a faster approach for the modelling stage might be carried out, in cases characterised by high similarity between the alternatives, by other formalisms. For example, in the case of a family of machines with small structural differences, a compound Petri net may be an appropriate formalism, not only for obtaining a small model but also for constructing the model in a productive way, by developing the common structure and particularizing the details of every alternative by means of a set of parameters and the associated set of values for them.

Analogously, the AAPN or the disjunctive coloured Petri net might lead to an easy modelling process when the ASC are obtained by different combinations of a set of subnets.

The last important feature of the formalisms that will be considered in this document is the availability of tools for modelling, analysing, and simulating the models.

In particular, the tools should permit including in the model the elements that allow representing the exclusiveness between alternatives: parameters in the case of compound Petri nets and guard functions for certain transitions in the case of the AAPN.

	Size rate	Modelling easiness	Practical tools
Set of alternative PN	Usually largest	Intuitive	No restrictions
Compound PN	Small with similar incidence matrices	Easy with similar incidence matrices	For parametric Petri nets
AAPN	Small with shared subnets	Easy with shared subnets	Allowing guards in transitions
DCPN	Small with shared subnets	Easy with shared subnets	For Colored Petri nets

**Table 1:** Summary of main characteristics of the formalisms presented in this document with regard to three key concepts.

Disjunctive coloured Petri nets require to model choice colours, which are supported by any coloured Petri net tool. A summary of the previous considerations is provided in Table 1.

## 6 Polytypic Sets of Exclusive Entities

This section is devoted to abstracting the feature of the Petri net formalisms able to represent a set of ASC. As a result of this process, a characterisation of all this kind of formalisms will be stated.

### 6.1 Motivation

All the formalisms presented in this paper as tools for modelling DES with ASC can be applied to the construction of disjunctive constraints in decision problems.

There is a different feature in every one of these formalisms to describe the exclusive nature of each alternative structural configuration. This feature is a set of exclusive entities, whose cardinality is the same as the number of ASC.

In the case of a set of alternative Petri nets, every pair of nets are mutually exclusive, while in the case of a compound Petri net, the set of exclusive entities is a set of feasible combination of values for the structural parameters of the net. The AAPN presents a set of choice variables and a disjunctive coloured Petri net includes a set of choice colours.

### 6.2 Definition

The examples that have been shown in this document include Petri net models with a monotypic set of exclusive entities, meaning that a single formalism has been

chosen for constructing the whole disjunctive constraint in the form of a Petri net [22].

Let us consider a DES  $D$ , whose structure is not completely defined. Let us consider that there are  $n$  ASC for  $D$ , able to determine completely the structure of  $D$ . It is possible, although not necessary in this context, to obtain a different alternative Petri net model for  $D$  from each one of the different structural configurations. As a result, it would be possible to obtain a set of  $n$  alternative Petri nets  $S_R = \{R_1, \dots, R_n\}$ .

#### Definition 5. Monotypic set of exclusive entities.

A monotypic set of exclusive entities related to a DES  $D$  is a set  $S_x = \{X_1, \dots, X_n\}$ , such that

i) The elements of  $S_x$  are exclusive, i.e. only one of them can be chosen as a consequence of a decision.

$$\forall i, j \in \mathbf{N}^*, i \neq j, 1 \leq i, j \leq n$$

$$ii) X_i \neq X_j$$

$$iii) \text{type}[X_i] = \text{type}[X_j].$$

iv)  $\exists f: S_x \rightarrow S_R$ , where  $S_R = \{R_1, \dots, R_n\}$  is a set of alternative Petri nets, feasible models of  $D$ , such that  $f$  is a bijection, meaning that  $\forall X_i \in S_x \exists! f(X_i) = R_i \in S_R$  and  $\forall R_i \in S_R \exists! f^{-1}(R_i) = X_i \in S_x$ .  $\square$

Nevertheless, it is also possible to combine different formalisms for modelling a given DES. The reason for following this strategy may arise from the conclusions presented in section 5, where a comparative of the advantages and drawbacks of the formalisms is performed. In fact, a set of alternative Petri nets can be decomposed into different subsets, whose elements might present differences regarding the similarity between the ASC of the DES. In this case, the associated set of exclusive entities is not a monotypic one anymore but a polytypic set [20], [18].

#### Definition 6. Polytypic set of exclusive entities.

A polytypic set of exclusive entities associated to a DES  $D$  is a set  $S_x^p = \{X_1, \dots, X_n\}$ , which verifies that

i) The elements of  $S_x^p$  are exclusive, i.e. only one of them can be chosen as a consequence of a decision.

$$ii) \forall i, j \in \mathbf{N}^*, i \neq j, 1 \leq i, j \leq n \text{ then } X_i \neq X_j.$$

iii)  $\exists S_x, S_x' \subseteq S_x^p$ , such that  $\forall X_i \in S_x, X_j \in S_x'$  it is verified that  $\text{type}[X_i] \neq \text{type}[X_j]$ .

iv)  $\exists f: S_x^p \rightarrow S_R$ , where  $S_R = \{R_1, \dots, R_n\}$  is a set of alternative Petri nets, feasible models of  $D$ , such that  $f$  is a bijection, meaning that  $\forall X_i \in S_x^p \exists! f(X_i) = R_i \in S_R$  and  $\forall R_i \in S_R \exists! f^{-1}(R_i) = X_i \in S_x^p$ .  $\square$

### 6.3 Examples

In [22] the modelling of a manufacturing facility with a number of alternative structural configurations is carried out by two monotypic set of exclusive entities in the form of a set of alternative Petri nets and a disjunctive coloured Petri net respectively.

Furthermore, [20] is a document devoted to exploring some modelling possibilities of the polytypic sets of exclusive entities. This reference presents an example, where the same model is transformed to be represented by three different formalism. In two of the representations a monotypic set of exclusive entities has been used, while in the third case, a polytypic set has been considered as a set of diverse alternative compound Petri nets.

The configuration of a set of exclusive entities as a polytypic one is the basis of the potential that this methodology presents for using distributed computation to speed up the solving methodology of the decision problems with disjunctive constraints. This topic has been addressed in [18].

## 7 Optimization with Petri Nets as Disjunctive Constraints

Section 1 discussed a general algorithm for decision making support based on simulation, which can be found extensively in the scientific literature. This algorithm is applicable under several approaches for cases where there is not any disjunctive constraint in PN form.

The introduction of formalisms able to model sets of exclusive entities requires the adaptation of the decision making methodology to the special features of a disjunctive constraint represented by one or several of these formalisms [17].

In fact, it is desirable not to modify the steps of the algorithm presented in the introduction, due to its success in diverse applications, as well as the existence of tools developed to apply this procedure. The main difference of the classic approach with the proposed methodology to be applied using the formalisms with exclusive entities (others than a set of alternative Petri nets) is to reduce the number of instances of decision problems to be solved, from  $k \in \mathbf{N}$ , where  $1 \leq k \leq n = \text{card}(S_R)$ , to just one. However, if several independent processors are available, this objective can be adapted [18].

In other words, instead of dodging the disjunctive constraint of the decision problem by choosing a subset of  $k$  promising alternative Petri nets and solving  $k$  inde-

pendent decision problems, a single decision problem is tackled with the disjunctive constraint represented in the form of a single Petri net with exclusive entities.

The only requisite to perform this reduction in the number of instances of the decision problem is to include in any solution to the problem a mechanism to choose one exclusive entity prior to the development of a simulation. This additional information has two purposes:

- a) The solution itself bears the necessary information to simulate the model of the system under a single structural configuration, thus, regarding the structure of the system, performing a deterministic simulation.
- b) Once the decision making problem has been solved, the solution(s) furnishes the decision maker with a suggestion on the most promising ASC.

As a conclusion, when applying this proposed methodology for decision making support, all the three drawbacks of the classic methodology, highlighted in section 2.1 have been overtaken [9].

## 8 Conclusions

In this document, a systematic approach for the description of disjunctive constraints represented by PN models is addressed. In particular, four formalisms have been presented: a set of alternative Petri nets, a compound Petri nets, an AAPN, and a disjunctive coloured Petri net.

All these formalisms have in common the inclusion of a mechanism to represent a set of exclusive entities, which are the main feature of the disjunctive constraint. Moreover, this approach allows the combination of different Petri net based formalisms to describe diverse parts of the same Petri net model, with the purpose of profiting from the nature of the system to be modelled and the features of the formalisms.

An application of this approach can be found in the statement of decision problems based on the simulation of the model of a system with ASC. This methodology proves that splitting the problem in a pre-selected number of subproblems may a less effective strategy. A decision making support problem with a set of ASC for the system of interest can be very common in a design process.

The research line presented in this document seems promising. However, there are open research questions that should be solved. On the one hand, more applications should be performed to get information on its suitability and success for the decision making in diverse fields and processes.

On the other hand, more effort should be devoted to the characterization of a given DES to decide which combination of formalisms to choose for describing it and how to minimise the size rate of the final model.

## References

- [1] Choi BK, Kang DH. Modeling and Simulation of Discrete Event Systems. Wiley. 2013. 432 p.
- [2] Cassandras CG, Lafortune S. Introduction to Discrete Event Systems. 2nd ed. Springer. 2008. 772 p.
- [3] Silva M. Half a century after Carl Adam Petri's Ph.D. thesis: A perspective on the field. Annual Reviews in Control, 2013; 37(2):191–219. doi: 10.1016/j.arcontrol.2013.09.001.
- [4] Silva M. Introducing Petri nets, In: Di Cesare, F. (editor). Practice of Petri Nets in Manufacturing, 1993, Ed. Chapman&Hall. 62 p.
- [5] Mújica-Mota M. Check-in allocation improvements through the use of a simulation–optimization approach. Transportation Research Part A: Policy and Practice, 2015; vol. 77: 320–335. doi: 10.1016/j.tra.2015.04.016
- [6] Mušić G. Schedule Optimization based on Coloured Petri Nets and Local Search, SNE Simulation Notes Europe, 2013; 23(1): 9 - 16. doi: 10.11128/sne.23.tn.10163.
- [7] Piera MA, Narciso M, Guasch A, Riera D. Optimization of Logistic and Manufacturing Systems through Simulation: A Colored Petri Net-Based Methodology. Simulation: Transactions of the Society for Modeling and Simulation International., 2004; 80(3): 121-129. doi: 10.1177/0037549704045045.
- [8] Baruwa OT, Piera MA, Guasch A. Deadlock-Free Scheduling Method for Flexible Manufacturing Systems Based on Timed Colored Petri Nets and Anytime Heuristic Search. Transactions on Systems, Man, and Cybernetics: Systems, IEEE, 2014, 45(5): 831 – 846. doi: 10.1109/TSMC.2014.2376471.
- [9] Latorre-Biel JI, Jiménez-Macías E. Efficient Methodology for High Level Decision Making on a Manufacturing Facility. In Proceedings of the 8th EUROSIM Congress on Modelling and Simulation. EUROSIM 2013, 2013 Sept, Cardiff (UK), p. 345 – 350. doi: 10.1109/EUROSIM.2013.68.
- [10] Zimmermann A, Freiheit J, Huck A. A Petri net based design engine for manufacturing systems. International Journal of Production Research, 2001; 39(2): 225-253. doi: 10.1080/00207540010004287.
- [11] Zhou M, Venkatesh K. Modelling, Simulation and Control of Flexible Manufacturing Systems. A Petri Net Approach. WS World Scientific, 1999, 428 p.
- [12] Tsinarakis GJ, Tsourveloudis NC, Valavanis KP. Petri Net Modeling of Routing and Operation Flexibility in Production Systems. In Proc. of the 13th Mediterranean Conf. on Control and Automation; 2005 Jun; Limassol, Cyprus, p.352-357. doi: 10.1109/2005.1467040.
- [13] Latorre-Biel JI, Jiménez-Macías E, Blanco-Fernández J, Martínez-Cámara E, Sáenz-Diez JC, Pérez-Parte M. Decision Support System, Based on the Paradigm of the Petri Nets, for the Design and Operation of a Dairy Plant. International Journal of Food Engineering. 2015; 11(6): 767–776. doi: 10.1515/ijfe-2015-0063.
- [14] Latorre-Biel JI, Jimenez-Macias E, Perez-Parte M. Sequence of decisions on discrete event systems modeled by Petri nets with structural alternative configurations. J. of Computational Science, 2014; 5(3): 387–394.
- [15] Latorre-Biel JI, Jiménez-Macías E, García-Alcaraz JL, Sáenz-Diez JC, Pérez-Parte M. Alternatives aggregation petri nets applied to modular models of discrete event systems. In Proceedings of the 27th European Modeling and Simulation Symposium. EMSS 2015, 2015 Sept; Bergeggi, Italy, p. 465- 471.
- [16] Latorre-Biel JI, Jimenez-Macias E, Perez-Parte M. The optimization problem based on alternatives aggregation Petri nets as models for industrial discrete event systems. Simulation: Transactions Of The Society For Modeling And Simulation International, 2013; 89(3):346-361. doi: 10.1177/0037549712464410.
- [17] Latorre-Biel JI, Jimenez-Macias E, Perez-Parte M, Blanco-Fernández J, Martínez-Cámara E. Control of Discrete Event Systems by means of Discrete Optimization and Disjunctive Colored PN: Application to Manufacturing Facilities. Abstract and Applied Analysis, 2014; 2014(3):1-16. doi: 10.1155/2014/821707.
- [18] Latorre-Biel JI, Jimenez-Macias E. Simulation-based optimization of discrete event systems with alternative structural configurations using distributed computation and the Petri net paradigm. Simulation: Trans. of the Society for Modeling and Simulation International, 2013; 89(11):1310-1334. doi: 10.1177/0037549713505761.
- [19] Zimmermann A, Rodríguez D, Silva M. A Two Phase Optimisation Method for Petri Net Models of Manufacturing Systems. Journal of Intelligent Manufacturing, 2001; 12(5):409-420. doi 10.1023/A:1012292102123.
- [20] Latorre-Biel JI, Pérez-Parte M, Sáenz-Diez JC, García-Alcaraz JL, Jiménez-Macías E. Mixing formalisms based on Petri nets for improving the efficiency of modelling and simulation of DES. In Proceedings of the 26th European Modeling and Simulation Symposium. EMSS 2014, 2014 sept; Bourdeaux, France.
- [21] Latorre Biel JI, Jiménez-Macías E, Pérez-Parte M. Control of Discrete Event Systems Modelled by Petri Nets. In Proceedings of the 7th EUROSIM Congress on Modelling and Simulation, 2010 Sept. Prague.
- [22] Latorre-Biel JI, Pérez-Parte M, Jiménez-Macías E. Disjunctive coloured Petri nets: a formalism for improving the applicability of CPN to the modeling of DES with alternative structural configurations. In Proceedings of the 26th European Modeling and Simulation Symposium. EMSS 2014, 2014 sept; Bourdeaux, France.