Methodology for Modeling, Parameter Estimation, and Validation of Powertrain Torsional Vibration

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Abstract. A vehicular powertrain is a lightly damped dynamic system that transfers the engine torque to the driving wheels through a number of inertias and elastic elements. Therefore, it is prone to vibrate and emit noise when disturbances are applied. Providing a methodology, for powertrain vibration modeling and simulation, is one of the key steps in various research topics in the field of automobile engineering. Verification of the engine crankshaft torsion and vibration model, as a subsystem of the powertrain, is proposed in this paper. This is achieved by constructing a rotational multi-body system in MATLAB and utilizing nonlinear least squares method for estimation of the model parameters. The simulated engine angular velocity is compared to the measured data, from a car, which shows a good agreement.

Introduction

There are different applications in automotive industry where powertrain vibration modeling is needed. Two of the more important cases are as follows:

- Passenger comfort is important for the customers, and consequently the car manufacturers. Powertrain dynamics is one of the main sources of noise, vibration, and harshness (NVH) inside the cars, and reducing its torsional vibration, to an acceptable level, is desirable. A typical powertrain NVH spectrum for a passenger car contains a considerable range of frequencies, from 2 Hz to 5000 Hz [1]. To achieve suitable ride quality, there is a need for better understanding of the dynamics, and a good model is a valuable instrument.
- To distinguish the effects of different excitation sources on the angular velocities of the powertrain parts, is not a trivial task. This causes difficulties for any type of miss-behavior detection. Examples of possible input disturbances are, combustion variation such as, misfire, cold start and cylinder variations; crankshaft torsional vibrations; road roughness; underinflated tires, etc. By using an appropriate powertrain model, and studying the distinct disturbances influences on the simulated outputs, such as angular velocities, it is possible to gain understanding and detection of undesirable modes. Examples of such applications of a model are misfire and underinflated tires, which are investigated by considering the simulated flywheel and driving wheel velocity signals, respectively [2-3].

Powertrain vibration can be modeled by torsional elastic elements. Basic models are linear lumped spring-massdamper systems which can be extended by adding details for the dynamics of the various parts [4]. Rabeih developed a 14-degrees-of-freedom lumped parameter model, from the engine to the driving wheels, for torsional vibration analysis of the powertrain with a four-cylinder engine and manual transmission [5]. The proposed model was good enough for simulating free, steady, and transient situations.

However, no experimental evaluations were done. Crowther used a 6-degrees-of-freedom model to perform numerical simulations for transient vibrations [6]. A powertrain test rig, that was used for measuring torsional vibration, was also presented. Furthermore, powertrain nonlinear torsional phenomena such as, gear backlash and a multi-stage nonlinear clutch, have been studied by Couderc et al., where an experimental test rig was also developed to verify the simulation results [7].

In all the mentioned works, the system parameters are approximate to the passenger cars and no details are provided. In contrast to the previous mentioned studies, the contribution of this paper contains the following three elements. Firstly, building a slightly different model (using dampings between all the inertias). Secondly, focusing on the parameter estimation procedure for the nonlinear engine block model by considering idling condition and using the measurements from a car. Thirdly, performing sensitivity analysis to study the behaviour of the system output with respect to changing of different parameters. This will provide the answer for the question, how important are different parameters in a model.

There is an enormous literature about parameter estimation methods and system identifcation for linear and nonlinear systems, e.g., [8-9]. The procedure for estimating unknown states and parameters of dynamical systems, from noisy measurements, consists of three main tasks [8]:

1. Three basic entities:
    a. Measurements of the inputs and the outputs.
    b. A model structure.
    c. A rule by which candidate models can be assessed using the measurements
2. Model validation which is done using separate data from the estimation data.
3. The system identification loop.

In this work, in-cylinder pressures, and the engine angular velocity, are measured for different working cycles at the idling condition, step 1.(a). Then, the engine-block torsional vibration model is constructed with unknown parameters which have physical interpretations, namely, spring, mass, and damper components, step 1.(b) in the above description. This is called gray-box model. Further, the nonlinear least squares is used as the rule to estimate the model unknown parameters, step 1.(c), by utilizing the experimental data for one working cycle, from step 1.(a). This is named estimation cycle. Finally, the estimated model is validated by simulating the system, at another cycle, and then comparing the engine angular velocity with the validation data, which is step 2. It is possible that the obtained model is not able to pass validation test, then it is necessary to go back and revise different steps of the procedure, which is step 3.

1 Engine-block Model

The proposed powertrain model is a nonlinear lumped parameter system consisting of rotating masses, friction, damping, and stiffness elements, which is shown in Figure 1. There exists different subsystems, i.e., engine block, transmission, final drive, and the wheels. The total friction in each subsystem is modeled by a number of dampings, which are connected to the ground for all the rotating masses. The modular structure of the model makes it flexible enough to add (remove) components with respect to different vehicle configurations such as front-wheel, rear-wheel, or four-wheel drive cars. In order to achieve better identification performance, the suggestion of this work is to disengage the clutch, i.e. idling condition, so that there is no connection between the flywheel and the transmission. Thus the model is simplified to the engine-block and consequently the delivered torque at the flywheel is zero, since, the engine friction will cancel out the produced torque by the cylinders.

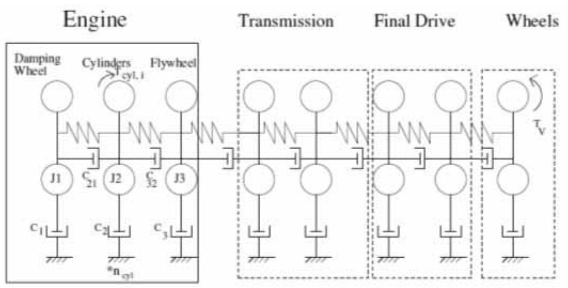


Figure 1: Powertrain model including engine block.

1.1 Mathematical modeling

According to Newton second law, the equations of motion for six different rotating masses, J_i, in the engine block which is a multiple-degrees-of-freedom system, are given by:

$$J_i\ddot{\theta}_i = C_{i+1,i}(\dot{\theta}_{i+1} - \dot{\theta}_i) - C_i\dot{\theta}_i + K_{i+1,i}(\theta_{i+1} - \theta_i), \text{ for } i = 1, \text{ Damping wheel} \quad (1)$$

$$J_i\ddot{\theta}_i = C_{i+1,i}(\dot{\theta}_{i+1} - \dot{\theta}_i) - C_i\dot{\theta}_i + K_{i+1,i}(\theta_{i+1} - \theta_i) - C_{i,i-1}(\dot{\theta}_i - \dot{\theta}_{i-1}) - K_{i,i-1}(\theta_i - \theta_{i-1}) + T_{cyl,i} \quad \text{for } i = 2,3,4,5, \text{ Four cylinders} \quad (2)$$

$$J_i\ddot{\theta}_i = -C_i\dot{\theta}_i - C_{i,i-1}(\dot{\theta}_i - \dot{\theta}_{i-1}) - K_{i,i-1}(\theta_i - \theta_{i-1}) \quad \text{for } i = 6, \text{ Flywheel} \quad (3)$$

There θ_i, $\dot{\theta}_i$, and, $\ddot{\theta}_i$ are angular position, angular velocity, and angular acceleration of the inertia at position i, respectively. Furthermore, C_i is the friction coefficient for element i, and $C_{i,i-1}$ and $K_{i,i-1}$ are the damping and stiffness coefficients, respectively, between two following inertias. The resulting torque, $T_{cyl,i}$, from cylinder i, which imposes the nonlinearity on the system, is calculated as a function of compression pressure force, $F_{c,i}$, and the piston mass force, $F_{p,i}$:

$$T_{cyl,i}(\tilde{\theta}_i) = \left(r\sin(\tilde{\theta}_i) + \frac{r^2 \sin(2\tilde{\theta}_i)}{2\sqrt{l^2 - r^2\sin^2(2\tilde{\theta}_i)}} \right) \times \left(F_{c,i}(\tilde{\theta}_i) + F_{p,i}(\tilde{\theta}_i)\right) \quad (4)$$

where $\tilde{\theta}_i = \theta_i + \delta\theta_i$, and $\delta\theta_i$ is the offset for each cylinder according to ignition order, here, 1-3-4-2. Also, r and l are crankshaft radius and connecting rod length, which are the geometrical characteristics of the cylinder model. More details on the equations and the model for the cylinder pressures and reciprocating torques can be found in Eriksson *et al*. [2].

1.2 State-space equations

The equations (1)-(3) can be transferred to the state-space form by performing two steps. First, utilizing the experimental data of the in-cylinder pressures, at idling, to obtain the delivered torque, $T_{cyl,i}(\tilde{\theta}_i)$, of cylinder i at different angular positions [2]. Second, defining new variables for angular velocities, $\dot{\theta}_i$, of each rotating mass, J_i. Therefore, there will be totally 12 states, including 6 angular positions, θ_i, and 6 angular velocities, $\dot{\theta}_i$. Then, the engine-block mathematical model in vector form, as a nonlinear ordinary differential equation, (ODE), can be written as follows:

$$\dot{\boldsymbol{x}} = \mathrm{f}(\boldsymbol{x}, \boldsymbol{p}), \quad y = \boldsymbol{x}(2) \quad (5)$$

where $\boldsymbol{x}$ is the state vector, y is the model output, which is damping wheel angular velocity $\dot{\theta}_2$, and $\boldsymbol{p}$ is the vector of unknown parameters. The complete powertrain model contains 42 unknown parameters. With the aid of symmetry and the proposed method of using idling conditions for parameters identification, the number of parameters, to be estimated in the engine-block, is reduced to 10. In Figure 1, 8 parameters are noted. The 2 remaining are damping between two cylinders, C_{xx}, and the piston reciprocating mass, m.

The vector of parameters, $\boldsymbol{p}$, can be categorized in two groups, i.e., 1) inertias and piston reciprocating mass, 2) friction coefficients and dampings. The crankshaft system is very stiff, therefore the stiffness coefficients are limited to high values and are hence not estimated.

2 Engine-block Parameter Estimation

One of the most well-known parameter estimation procedures, in nonlinear systems, is to write the estimation problem as a nonlinear optimization problem, where the goal is to minimize the difference between the predicted output, from the model, and the observations. The nonlinear least squares method, which is applied in this paper, is a commonly used optimization formulation. It searches the parameters values which minimize the squared errors.

2.1 Nonlinear least squares (NLS) formulation of the problem

A nonlinear regression model is described by an equation of the form:

$$z_i = g(\boldsymbol{u}_i, \boldsymbol{p}) + \varepsilon_i \quad (6)$$

where z_i denotes the observations, $\boldsymbol{p}$ is the set of unknown parameters that are to be estimated, $\boldsymbol{u}_i$ is a $1 \times k$ vector of known values, and g is a nonlinear function [10]. The least squares estimate is computed by minimizing the following objective function:

$$V_N(\boldsymbol{p}) = \sum_{i=1}^{i=N} (z_i - g(\boldsymbol{u}_i, \boldsymbol{p}))^2 = \sum_{i=1}^{i=N} \varepsilon_i(\boldsymbol{p})^2 \quad (7)$$

where, for the engine block estimation problem, $\varepsilon_i(\boldsymbol{p})$ is the discrepancy between the simulated and the measured damping wheel angular velocity. In general, the solution of the above mentioned nonlinear minimization problem, is not available analytically, thus numerical nonlinear optimization algorithms are used [10].

2.2 Forward sensitivity analysis (FSA)

Sensitivity analysis is a procedure for studying the influence of a parameter value perturbation on the model behaviour. Forward sensitivity analysis is a local method [11] which can be performed by introducing the following initial value problem with n states and m parameters $\boldsymbol{p}$:

$$\dot{\boldsymbol{x}} = \mathrm{f}(t, \boldsymbol{x}, \boldsymbol{p}), \quad y = \boldsymbol{x}(2) \tag{8}$$

and the augmented system with sensitivity equations is given as:

$$\dot{\boldsymbol{w}} = \begin{pmatrix} \mathrm{f}(t, \boldsymbol{x}, \boldsymbol{p}) \\ \boldsymbol{P}'_i \end{pmatrix}, \quad \boldsymbol{w} = \begin{pmatrix} \boldsymbol{x} \\ \boldsymbol{P}_i \end{pmatrix}, \quad \boldsymbol{P}'_i = \frac{\partial \mathrm{f}}{\partial \boldsymbol{x}} \boldsymbol{P}_i + \frac{\partial \mathrm{f}}{\partial \boldsymbol{p}}, \boldsymbol{P}(0) = 0 \tag{9}$$

where $\boldsymbol{P} = \frac{\partial x}{\partial p}$ is an $n \times m$ matrix function and $\boldsymbol{P}_i$ points to column i in matrix $\boldsymbol{P}$ which corresponds to one parameter in $\boldsymbol{p}$.

In this paper, FSA is to investigate the engine-block behaviour for small changes of the parameters near estimated values found by NLS.

3 Results and Discussion

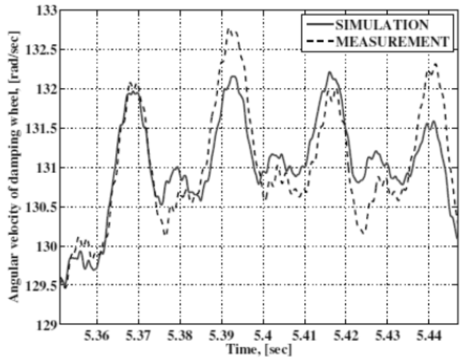
3.1 Estimation results using numerical solver

The `lsoptim` is a package, which is written in-MATLAB by Eriksson [12], for solving non-linear unconstrained least squares optimization problems. The solver uses the Levenberg Marquardt method, as the optimization numerical algorithm. It is second order and thus has good local convergence properties. The iterative optimization, which is applied in this solver, is based on three stages:

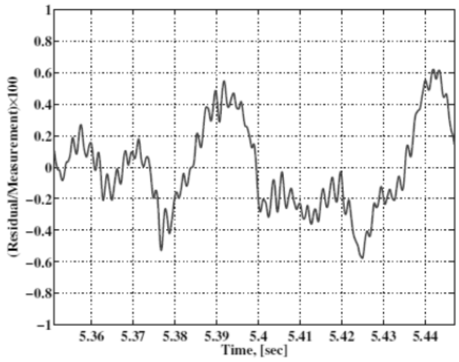
- Start: providing an initial guess for parameters which are to be estimated.
- Iteration: simulation of the ODE in (5) to find the damping wheel angular velocity, which is one of the states in the state vector, $\boldsymbol{x}$. Later, the residual, $\varepsilon_i(\boldsymbol{p})$, can be determined by subtracting the simulated value of this state from the measured data. The residual is used in each iteration to define the direction for updating the set of parameters.
- Stop: The search algorithm will be stopped by a criterion based on the difference between the values of the objective function in two following iterations. In other words, the final iteration will not improve the objective function more than a certain degree. It is worth to mention that lsoptim only converges to a local minimum (which might be the global minimum as well).

In Figure 2(a), the simulated and the measured damping wheel angular velocities, are compared by applying the estimated parameters provided by the numerical solver.

High resolution measurements, for a 4-cylinder car, with angular resolution of 0.5 degree, are used. It is seen the model is not only able to capture the main frequency from the engine and follow the trend, but also, it resembles the measurement data at higher frequencies. The ratio of the residual value, $\varepsilon_i(\boldsymbol{p})$, over the real data, in each time sample i, is shown in Figure 2(b). It provides how far the model result is from the reality, which shows the maximum value of $\sim 0.6\%$, when the engine mean angular velocity is approximately 130 rad/sec.



(a): Damping wheel angular velocity, measurement vs. simulation.

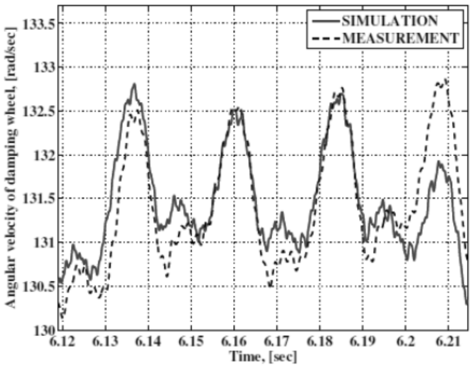


(b): The ratio of the residual value $\varepsilon_i(\boldsymbol{p})$ over the real data, in each time sample i.

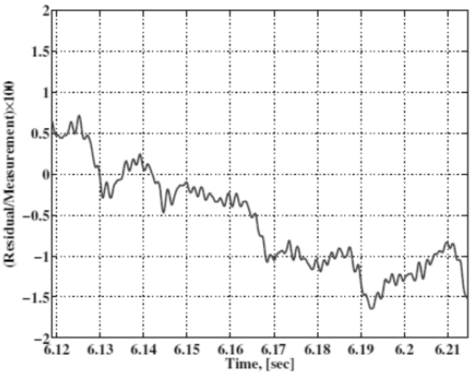
Figure 2: Comparison between simulation and empirical data for *estimation cycle*.

3.2 Model validation

According to the Introduction, the second step in the procedure of the estimation, is model validation, in which the model output is compared with validation data. In other words, for the engine model, the damping wheel angular velocity observations, at validation cycle, is plotted against the simulated results. Therefore, it is determined whether the estimated model accurately captures the system dynamics or not.



(a): Damping wheel angular velocity, measurement vs. simulation.



(b): The ratio of the residual value $\varepsilon_i(\boldsymbol{p})$ over the real data, in each time sample i.

Figure 3: Comparison between simulation and empirical data for *validatioin cycle*.

Figure 3(a) shows the measured versus simulated data plot for validation data, and Figure 3(b) shows the ratio of the residual value over the real data, in each time sample. It is seen that there exists a good agreement between the validation data and the output from the model, which is a good evaluation for the estimated model.

3.3 Results of sensitivity analysis

Here, the effects of the 10 parameters perturbations, mentioned in Section 1.2, are considered. The primary system equations, given in (8), were written in Mathematica and then the sensitivity equations were computed. The final ODE system (9) has been solved in MATLAB.

Figure 4 shows the sensitivities of the damping wheel angular velocity, (rad/sec), to the three friction coefficients, $C_1 - C_3$, and to the three damping coefficients, C_{21}, C_{32}, and C_{xx}.

Looking at the values of all the plots on the vertical axis (rad/sec), it is understood that the sensitivities to friction coefficients are in the same level and significantly higher than the sensitivities to the damping coefficients. Possible interpretation is that, friction coefficients are important to follow the trend and changing them has a high influence on increasing or decreasing the engine velocity (rad/sec). However, from the basic vibration knowledge, it is known that the damping coefficients only control the amount of oscillations and have nothing to do with the mean value of the angular velocity. There also exist interpretations for two different shapes of the sensitivities, described as follows:

- The reducing shape for the sensitivities of the engine angular velocity, (rad/sec), to the three friction coefficients, $C_1 - C_3$, is the result of friction growth. In other words, by increasing friction coefficient, the system is slowed down, and the slope, $\dot{\omega}$, can be obtained by the relation $\Delta C\omega \approx -(\sum J)\dot{\omega}$, in which ω is the current angular velocity of the system and $\sum J$ is the sum of the inertias in the model. The reduction rate, which is found from this relation, is the same as the one seen in the sensitivity plot. The slope for C_2 is four times of the other coefficients, namely, C_1 and C_3. The reason is that C_2 is repeated for the four cylinders. Therefore, by perturbation of C_2, all the four cylinders frictions will be perturbed.
- The damping coefficients will not alter the mean value of the output, and furthermore, it is seen from the plots that after a while, the amplitude of the oscillations becomes lower since the damping coefficients are perturbed.

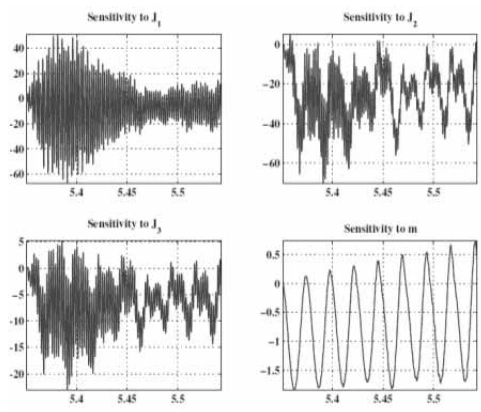


Figure 4: Sensitivities of the damping wheel angular velocity to 3 friction coefficients, $C_1 - C_3$, and 3 damping coefficients, C_{21}, C_{32}, and C_{xx}.

Figure 5 contains the sensitivities of the damping wheel angular velocity, (rad/sec), to the three inertias, i.e., damping wheel J_1, each cylinder J_2, flywheel J_3, and to the piston reciprocating mass, m. The inertias are important for catching the amplitudes of the main frequencies in the system. This is exactly what is seen from the shape of the sensitivities to $J_1 - J_3$. The oscillatory motion of the reciprocating mass of the piston adds an oscillatory torque to the pressure torque. This causes oscillations in the angular velocity of the engine as well, which is also presented in the sensitivity plot.

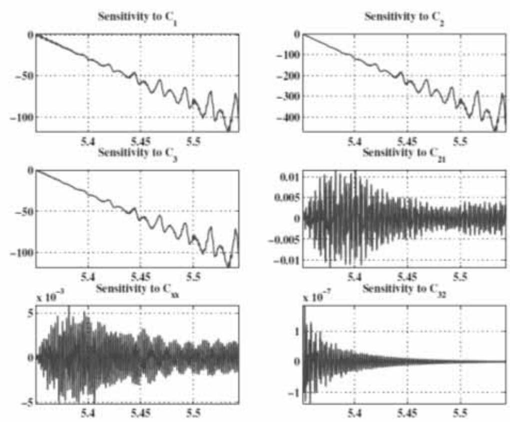


Figure 5: Sensitivities of the engine angular velocity to 3 inertias, i.e., damping wheel J_1, each cylinder J_2, flywheel J_3, and to the piston reciprocating mass m.

4 Conclusion

A powertrain model, suitable for studying torsional vibrations, was presented. The model is in modular form, and thus flexible enough, to be adapted for various powertrains structures, i.e., front-wheel, rear-wheel, and four-wheel drive systems with different number of cylinders. In order to get better parameter estimation performance, for the engine-block, the idling situation was considered and the clutch was disengaged. Therefore, the number of parameters to be estimated decreased to 10. The results from the model for damping wheel angular velocity, using the estimated parameters, validated versus the validation measured data, which showed significant similarity. This proves the ability of the model to resemble the main behaviours of the real system.

Furthermore, the performed sensitivity analysis was helpful to understand the importance of different parameters in the model, besides how the model responses to perturbation of these parameters.

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