Simulation of Heat Radiation Asymmetry with Maple

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Abstract. A new method was developed for determination of radiated temperature asymmetry. Application of the method resulted in more accurate data, where the plane dividing the two half spaces of the surface element at the test point separates the thermally active (coldest and warmest) surfaces in all cases and determines the irradiation factor through drawing. The present method of asymmetry calculation enables a more accurate calculation of the 'one side radiation asymmetry' parameter and provides a further characteristic parameter of the comfort of rooms in case of cold and warm surfaces being within the same plane.

Introduction

Former calculation procedures of radiation temperature asymmetry are only of approximate validity because of the fixed, i.e. vertical or horizontal position of the plane separating the two half spaces and because of the fact, that in practical calculations the room terminating surfaces beyond those being thermally active and their temperatures are taken into account with their average values [1].

A new method has been developed for determination of radiation temperature asymmetry "giving a result more closer to the reality", where the plane dividing the two half spaces of the surface element at the test point separates the thermally active (coldest and warmest) surfaces in all cases and determines the irradiation factor through drawing [2].

The present method of asymmetry calculation enables a more accurate calculation of the "one side radiation asymmetry" parameter defined by us and being a further characteristic parameter of the comfort of rooms in case of cold and warm surfaces being within the same plane.

Using the Maple computer algebra system there are several possibilities to simulate the radiation asymmetry values in space.

1 Educational Aspects

In the engineering education the students always ask about the usefulness of learning Mathematics. The question is usually justifiable because of the themes of basic Mathematics courses. The lack of time and the weak pre-education of the freshmen students only a few engineering applications are mentioned during the main courses. The solution of a complex engineering problem needs the knowledge of those kinds of – mainly numerical – methods, which are not taught in the courses of Mathematics.

The next model uses only a very few mathematical concepts. These concepts are taught in the first semester of all engineering courses. If there are any possibilities to get to know the elements of a computer algebra system, the simulation of heat radiation asymmetry is a good example to bring Mathematics closer to the engineering students.

2 Problem Definition

2.1 Analytical description

The radiation temperature asymmetry in a given room, in a given test point is the difference of the radiation temperature of the two half spaces, where the space is separated into two half spaces depending the shape and the place of the test body:

$$\Delta t_a = \left| t_{rad,1} - t_{rad,2} \right| \qquad (1)$$

where Δt_a is the radiation asymmetry (^{o}C), $t_{rad,i}$, is the radiation temperature of the *i*-th hemisphere (^{o}C).

The radiation temperature is the temperature of the surrounding surfaces at which the irradiative heat exchange of a body is the same, as the irradiative heat exchange of the different platforms.

The equation of the radiation temperature for a half spaces as follows:

$$t_{rad,i} = \sqrt[4]{\sum_{j=1}^{n} \varphi_{i,j} \cdot T_j^4} - 273 \qquad (2)$$

where $\varphi_{i,j}$ is the irradiation coefficient of i-th surface toward j-th surface, n is the number of surfaces in the half sphere, T_j is the temperature of the j-th surface in K.
For the calculation of radiation temperature asymmetry we have to determine the irradiation coefficients of the different boundary surfaces with different temperature from complicated equations.

In the case of different surface temperatures carrying out the calculations is very complicated, and if we get into account the great number of test points in a room, such as this procedure is rather time-consuming.

2.2 Geometrical constructional process

This constructional process uses the hemisphere shape space above the elemental surface to determine the irradiation coefficient (Fig. 1).

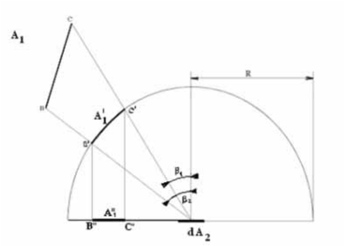


Figure 1: Determination of the irradiation coefficient of dA_2 elemental surface to A_1 surface in a two dimensional model.

In the first step in this process we generate a so called radiation hemisphere above the elemental surface, which is a semicircle in the two dimensional model. The dA_2 elemental surface is the geometrical centre of the basic surface. The radius of the sphere is arbitrary, in our calculation is equal to 1. In the next step we project the A_1 surface across the semicircle into the basic plane.

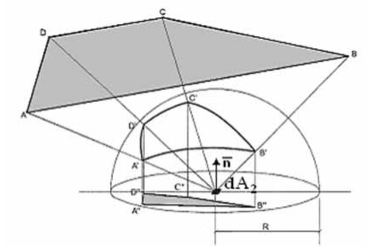


Figure 2: The dA_2 elemental surface irradiation coefficient determination in the three dimensional model.

The geometrical constructional process for determination of radiation coefficient in three dimensional cases is shown on the Fig. 2. The arbitrary corner elements of a room are projected to the hemisphere first, and the intersections and arcs are projected orthogonally to the circle shape basic surface in the next step.

3 Application of Geometrical Construction Process

We used the geometrical process in a special case, where the plane, which divides the space, divides the two surfaces with extreme temperature as well.

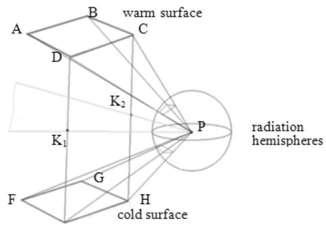


Figure 3: The plane which divided the space into two half spaces.

The plane which divides the space into two half spaces is defined by the observation point P and the K_1 and K_2 middle points of EI and CH sections.

Our goal was to develop a computer simulation, which can determine and visualise the radiation asymmetry values. The used Maple computer algebra system is able to visualise the steps of the construction, to visualise the results and calculate the irradiation coefficients in as many points as we want.

For the calculation we used only the elements of elementary vector algebra (equation of plane, line, determination of intersection of line and plane, equation of the sphere, territory of the rectangle).
For modelling mathematically the procedure, the marks, what will be used in the following part are:
a,b,c,d,e,f,g,h,p are the radius vectors of *A,B,C,D,E,F,G,H,P* points, respectively.
The steps of the process as follows,

- Determination of plane equation through the K_1, K_2, P points:
$$\boldsymbol{n}(\boldsymbol{r}-\boldsymbol{p}) = 0 \qquad (3)$$

where $\boldsymbol{n} = \overrightarrow{PK_1} \times \overrightarrow{PK_2}$ and $\boldsymbol{r}$ is the radius vector of the arbitrary point of the plane. The equation of the sphere with radius R, and centre P:

$$(\boldsymbol{r}-\boldsymbol{p})^2 = R^2 \qquad (4)$$

- Projection of the cold and warm active surfaces and the boundary walls to the sphere ($A'B'C'D'$ points): For A' point firstly the equation of AP line is written down:

$$\boldsymbol{r} = \boldsymbol{p} + t(\boldsymbol{a} - \boldsymbol{p}),\ t \in \Re \qquad (5)$$

The point A' is the intersection of (4) and (5).

- The spherical projected points projection to the K_1K_2P plane ($A''B''C''D''$ points):

Such as A'' point it is the intersection of the plane (3) and line, which is orthogonal to this plane and go through to A'.

- Determination of the $\varphi = \dfrac{area(A''B''C''D'')}{R^2\pi}$ values.
- Calculation of the $t_{rad,i} = \sqrt[4]{\sum_{j=1}^{n} \varphi_{i,j} \cdot T_j^4} - 273$ and the $\Delta t_a = \left| t_{rad,1} - t_{rad,2} \right|$ values.

Using the Maple computer algebra system we simulate the radiation asymmetry values in space, in three different cases.

3.1 Thermally active surfaces in the same plane – radiator under the window

In arbitrary observation points of a room we can calculate all irradiation coefficients, not only for the thermally active surfaces, but for all boundary surfaces as well.

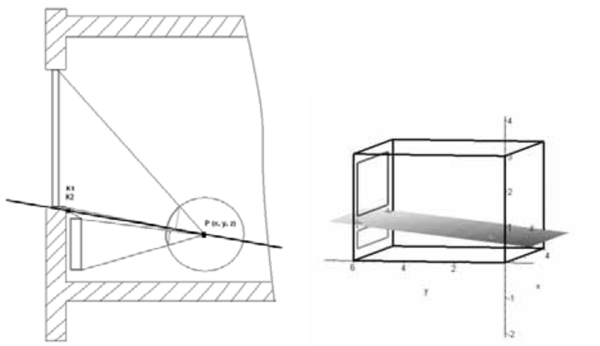


Figure 4. Observation plane in the case of radiator under the window.

The boundary values where the calculations were carried out were as follows: the temperature of the warm surface was 36 OC, the cold surface had 12 oC, and the walls had 18 oC. The sizes of the room in meter were x=4, y=6, z=3. The area of the warm surface (red rectangle) was 2.9x0.5 m2, the cold surface (blue rectangle) was 2.9x1.5 m2.

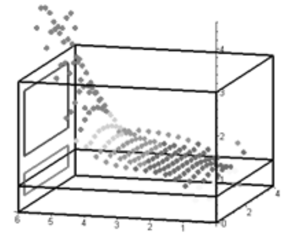


Figure 5. The radiation temperature asymmetry values in 0.6 m height.

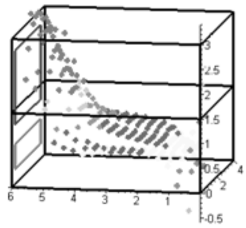


Figure 6. The radiation temperature asymmetry values in 1.5 m height.

In the Figure 5, Figure 6, and the Table 1 are demonstrated, that closer to the active surfaces the asymmetry values increase. The values are in ^{O}C. For humans these high values mean discomfort feeling. From this reason, it is not useful to place the bed or the desk close to the window.

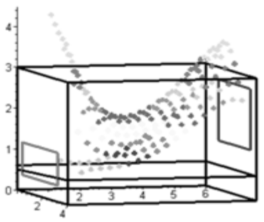
			Δt_a (°C)	z=0.6		
x	y=0.9	y=1.8	y=2.4	y=3.6	y=4.8	y=5.7
0.6	2.11	5.43	5.84	6.72	12.50	24.73
1.2	4.05	5.16	5.71	7.31	13.88	26.70
1.8	4.87	5.18	5.60	7.48	15.34	24.06
2.4	4.66	5.15	5.59	7.38	14.41	22.42
3.0	3.08	5.02	5.65	6.96	12.12	23.72
3.6	6.21	5.11	6.25	7.16	9.40	16.63

Table 1. Each figure and table caption is to be put below the figure, typeset like this caption. Insert each table *inline* and compose good, comprehensive captions.

3.2 Thermally active, parallel surfaces

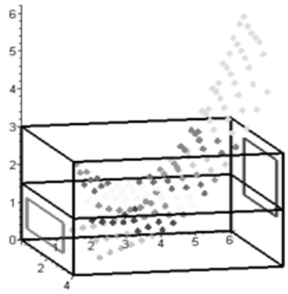
In this case the thermally active, parallel surfaces are on the opposite side (cold glass wall and wall-heating on winter time or warm glass and cooler on summer time).

In Fig. 7. and 8. it is shown that the absolute values are much more smaller, then it were in the first case, where the active surfaces were at the same plane. It is the reason, why the window and the cooler are usually on the opposite side of a room. The most comfortable place is in the middle of the room.



turquise	$3.3 \le \Delta t_a < 3.7$
red	$3.0 \le \Delta t_a < 3.3$
pink	$2.7 \le \Delta t_a < 3.0$
kheki	$2.4 \le \Delta t_a < 2.7$
orange	$2.1 \le \Delta t_a < 2.4$
yellow	$1.8 \le \Delta t_a < 2.1$
magenta	$1.5 \le \Delta t_a < 1.8$
blue	$1.2 \le \Delta t_a < 1.5$
acquamarine	$0.7 \le \Delta t_a < 1.2$

Figure 7: The radiation temperature asymmetry values in case of parallel active surfaces in 0.6 m height.

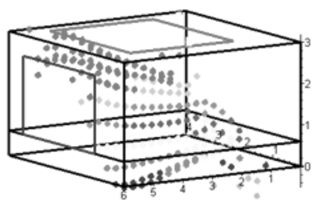


turquise	$3.0 \le \Delta t_a < 6.2$
red	$2.7 \le \Delta t_a < 3.0$
pink	$2.4 \le \Delta t_a < 2.7$
kheki	$2.1 \le \Delta t_a < 2.4$
orange	$1.8 \le \Delta t_a < 2.1$
yellow	$1.5 \le \Delta t_a < 1.8$
magenta	$1.2 \le \Delta t_a < 1.5$
blue	$0.9 \le \Delta t_a < 1.2$
acquamarine	$0.6 \le \Delta t_a < 0.9$
gray	$0.1 \le \Delta t_a < 0.6$

Figure 8: The radiation temperature asymmetry values in case of in case of parallel active surfaces in 1.5 m height.

3.3 Thermally active surfaces in orthogonal arrangements

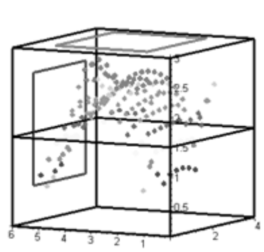
In this case warm (heating) ceiling or floor and cold (cooler) window (winter period), or cold ceiling or floor and warm window (summer period) are in the studied room (Fig. 9, 10).



red	$15.0 \le \Delta t_a < 18.0$
pink	$12.5 \le \Delta t_a < 15.0$
kheki	$8.0 \le \Delta t_a < 12.5$
orange	$5.0 \le \Delta t_a < 8.0$
yellow	$2.5 \le \Delta t_a < 5.0$
magenta	$1.5 \le \Delta t_a < 2.5$
blue	$0.9 \le \Delta t_a < 1.5$
acquamarine	$0.6 \le \Delta t_a < 0.9$

Figure 9: The radiation temperature asymmetry values in case of orthogonal arrangement in 0.6 m height. The values are between 0.6 and 18 ^{O}C.

It is interesting to observe, that the values and the differences in Fig. 10 are the highest one, nevertheless the floor heating nowadays are very popular.



red	$15 \le \Delta t_a < 53$
pink	$13.5 \le \Delta t_a < 15$
kheki	$13.0 \le \Delta t_a < 13.5$
orange	$10.5 \le \Delta t_a < 13.0$
yellow	$9.0 \le \Delta t_a < 10.5$
magenta	$7.5 \le \Delta t_a < 9.0$
blue	$6.0 \le \Delta t_a < 7.5$
acquamarine	$0.6 \le \Delta t_a < 6.0$

Figure 10: The radiation temperature asymmetry values in case of orthogonal arrangement in 1.5 m height. The values are between 0.6 and 50.3 ^{O}C.

4 Conclusion

According to the newest data of literature [4], [5], [6] humans feel unpleasant the asymmetrical radiation, when the differences of the heat stream on different part of the body are higher than 35-40 W/m^2. When the difference between the temperatures of the surfaces (ceiling, walls, windows) are higher than 5-6 K, there are a danger of discomfort feeling.

Our simulations are useable for house planning. The present values are somewhat different from the values, which were calculated the earlier used methods [3]. It does not cause any problem, because we can standardize the values, and for planning the trends, the fluctuations are interesting for the comfort feeling.

With help of this simulation there are possibilities to determine the zones in a room, where the humans feel themselves much more comfortable.

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