# Application of the Fourier Analysis for the Validation and Optimisation of Discrete Event Models

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Abstract. Logistic systems can be designed as pushsystems or pull-systems. In a pull-system, one parameter, e.g. the stock items have to be closed-loop controlled. It will be shown that the closed-loop controlled model can work in an unstable manner like a 'logistic oscillating circuit'. This behaviour is a part of the bullwhip effect, which occurs typically in supply-chains. In comparison with a closed-loop controlled electronic system, the elements of a logistic system have relatively long dead times. In discrete event simulation, there is no method to optimally calculate the parameters of the system also taking account of the dead times. Furthermore, there is no validation technique to date, which can determine the unstable behaviour of a system. In many different areas of engineering, the FFT analysis is a frequently used method. It will be shown that the FFT analysis is a suitable method to determine the unstable behaviour of discrete event simulation models. However, FFT analysis is only a method to determine the unstable behaviour; the elimination can only be done by trial and error.

### Introduction

Discrete event simulation is a frequently used method for designing new, complex production systems. The advantage of this method is that all elements of the system can be described with only a few parameters. Objectives of such a simulation study are to obtain knowledge of, for example, cycle times or utilisation of the machinery. There are numerous commercially available programs for this.

The execution of a complete simulation study is described in VDI 3633 [4]. An important point within a simulation study is the validation of the model and the results. There is no generally applicable guideline for this. Depending on the particular project, different methods can be applied. Table 1 presents a summary of all methods according Rabe et al [3].

## 1 Characteristics of a Closed-loop Controlled System

Push-systems are often used for the management of a production system. These systems are producing goods without a customer order. Therefore, the stock in such a system is relatively high. To reduce stock, a pull-system is the better solution. Here, the production only starts on demand. The higher the demand, the faster is the production rate. These systems are closed-loop controlled. These closed-loop controlled systems can of course also be designed using discrete event simulation. As there is usually no special module for this, the controller has to be designed using a special program code.

First, it is necessary to point out the differences between a logistics closed-loop controlled system and closed-loop controlled systems in for example automation technology. In the automation technology, the systems are designed using electronic modules. The signals in these systems are therefore electric currents or voltages. Therefore the speed of the signals is practically infinite.

In contrast, the speed of the signal in a logistics system is very low. The speed of a conveyor is low or the processing time on a production machine is high. Thus, the speed of the signal which is connected with the flow of objects through the production system is very low. Described in terms of control technology, the time difference between the entry of an object or signal to a machine and the exit is the dead time.

It is generally known that dead times in a closed-loop controlled system can result in oscillations and thus unstable systems. There are methods in control technology to design the parameters of a system so that these oscillations can be prevented. These methods could not be applied in the discrete event simulation due to missing mathematical relationship between the system parameters.

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	Target description	Task specification	Concept model	Formal model	Executable model	Simulation results	Basic Data	Processed Data
Animation					Х	Х		
Review	х	Х	Х	Х	Х	Х	Х	Х
Dimensional Consistency test				Х	х	Х	Х	Х
Event validity test					Х			
Fixed value test				Х	Х	Х		
Extreme condition test				Х	х	х		Х
Monitoring					х	Х		Х
Desk checking	Х	х	Х	Х	х	Х	Х	Х
Sensitivity Analysis					х	Х		Х
Statistical Techniques					х	Х	Х	Х
Structural walkthrough	Х	Х	Х	Х	Х	Х	Х	Х
Internal validity test					Х	Х		
Sub model testing			Х	Х	Х			
Trace analysis					Х			
Turing test					Х			
Cause-effect graph			Х	Х	Х			
Face validity	Х	Х	Х	Х	х	Х	Х	Х
Predicitve validation					Х			
Comparison to other models					Х	Х		
Historical data validation					Х			

Table 1. Validation methods (according to Rabe et al.).

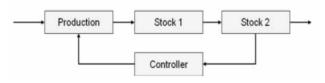


Figure 1. Simple closed-loop controlled production system.

# 2 Design of an Unstable Closed-loop Controlled System

First, it is necessary to demonstrate, that unstable behaviour can also occur in logistics systems. A very simple model will be designed for this (Figure 1): The production system delivers the goods to a stock 1. This stock has a constant storage time, e.g. for cooling. Then the items are stored in a stock 2, e.g. for distribution.

The stock output rate is constant. The difference between the current stock and the target stock is the parameter which controls the production cycle time. The higher the difference, the faster is the production (Figure 1). The system is designed with the following parameters:

- The cycle time of the production is closed-loop controlled
- The production has an infinite capacity
- The storage time of stock 1 is 1 day
- The target stock is 15 units for stock 2
- Both stocks have an infinite capacity
- The stock output rate of stock 2 is constant at 1 unit/4.8 h

This system is closed-loop controlled by a controller with the following characteristics:

- The current stock is recorded once per day
- The cycle time of the production is calculated with a delay of one half day
- The difference of the stock is fed into the production system on the next day

A discrete event model is created using these values (Figure 2). A similar model is presented in Barbey [1].

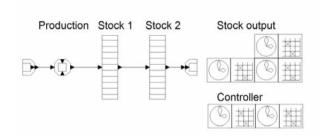
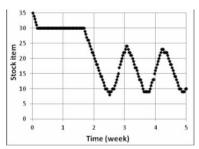


Figure 2. Discrete event model designed with DOSIMIS.

### 3 Validation and Results

In the following diagram the results for the stock are recorded over a period of 5 weeks. At the very beginning, the controller is switched off.



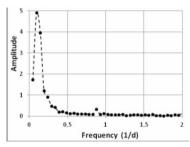


Figure 3. Stock of the first model.

**Figure 4.** FFT analysis with Excel for the first model.

The stock has a constant figure of 30 units here. After approx. two weeks, the controller is switched on. Then the stock has on average the target value. However, it is overlaid with an oscillation with an amplitude of 7 units (Figure 3).

This model demonstrates that unstable behaviour in logistics systems can occur if an unsuitable set of parameters is applied. All input parameters are constant, but the stock is only constant on average.

The model is acting like a 'logistic oscillating circuit'. This behaviour is a part of the bullwhip effect, which is a typical behaviour in supply-chains.

If unstable behaviour can be produced in one model, then it is also possible to produce unstable behaviour in any other model. Therefore, validation of the model is an important issue.

According to Table 1, there are many validation methods which can be applied. However, only the reviewing of the results is suitable for this model. However, this model has been designed as simple as possible with the only objective of indicating an error: in this case unstable behaviour. It is also obvious that this error will be found with the selected validation methods. However, the models are generally much more complex and the unstable behaviour is thus possibly not immediately visible.

All the methods in Table 1 can find almost any kind of error in discrete simulation models, but they are not suitable for detecting unstable behaviour. Therefore, a new validation method must be found. In many areas of engineering, the Fourier analysis is a frequently used method to analyse dynamic processes. This method states that each periodic signal can be compiled from a sum of sine and cosine-functions. The Fast-Fourier-Transform (FFT) is a special form of the Fourier analysis. The mathematical theory for this method is described in Brigham [2].

The Fourier analysis is initially applied to the described model. The results of the Fourier analysis which have been produced here using Excel are shown in Figure 4. The frequency range was analysed up to a frequency of 2 1/d. There is only one peak at a frequency of approx. 0.12 1/d. The same result can be obtained from the consideration of Figure 3. This frequency is the characteristic frequency of the un-

stable system. The application of the Fourier analysis to this simple model demonstrates that this method is suitable for the validation of discrete event models.

Chapter 2 illustrates the application of a new validation method to a very simple model. Normally, models are significantly more complex. Therefore, this simple model will be somewhat modified in the following.

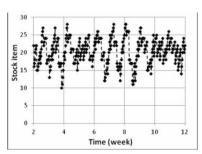
Two parameters are changed in the next model:

- The current stock is recorded once per day and the cycle time of the production is calculated immediately
- There is an additional stock output each 1.4d. The additional output is a random number between 4 and 8.

The results are shown in Figure 5. The result still seems to be periodic. There must be a periodic part in, because the additional stock output is periodic. But there is no method according to Table 1 in this case which detects the unstable behaviour of the model. The unstable behaviour is also is not visible in Figure 5. Therefore, the Fourier analysis is now applied to these results. The diagram in Figure 6 shows some typical characteristics of the model.

There is one peak at a frequency of 0.7 1/d. This is related to the additional output which has exactly this frequency. A second peak is at a frequency of 1.4 1/d which is double the frequency of the stock output. An extension of the frequency range would show additional peaks at all frequency multiples of the output. However, the largest peak is at a frequency of 0.15 1/d. This frequency cannot be explained by the additional output; it is related to the unstable behaviour of the model itself.

Using the frequency analysis, it is possible to determine the unstable behaviour of a model. For this, each characteristic frequency has to be compared with the parameters of the model. In this case, the additional output is a parameter which produces a periodic signal. Therefore, this frequency and its multiples can be excluded.



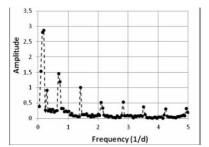


Figure 5. Stock with additional output.

**Figure 6**. FFT analysis with an additional output.

All other frequencies which could not be explained by the parameters of the model show an unstable behaviour of the model.

However, the unstable behaviour cannot be eliminated using the Fourier analysis as there is no functional relationship between the model parameters.

The elimination of the unstable behaviour can only be realised by trail and error. A variation of the model parameters and a following Fourier analysis shows whether the results are satisfactory.

## 4 Optimisation

Optimisation is always project-specific. Each model has a specific set of parameters which can be changed individually. The model is now only used to demonstrate how unstable behaviour can be eliminated by using the Fourier analysis.

In the original model, the controller was designed so that the difference in the stock is supplied by the transport system to the production system on the next day. Now the controller will be modified so that only one third of the difference will be supplied to the production system on the next day. Figure 7 shows the stock with the modified controller. Compared with Figure 5, the fluctuation of the stock is reduced. Figure 8 shows the Fourier analysis for this.

The peak at a frequency of 0.15 1/d is clearly reduced but still exists. All other peaks caused by the additional stock output are still there. An additional simulation with this model without the additional output shows the improvement actually achieved (Figure 9).

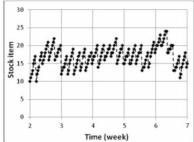
Figure 3 shows a permanent oscillation after the controller is switched on. In contrast, Figure 9 only shows one oscillation. The signal then has a con-

stant value of 15 and the bullwhip-effect is reduced. This demonstrates an improvement, but the optimum has not yet been reached. Therefore, further variation of the parameters has to be done. This procedure shows that the Fourier analysis is suitable for finding the instability of a model and optimising it afterwards.

#### References

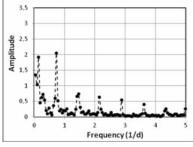
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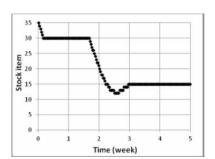


Time (week)

Figure 7. Stock with the modified controller.



**Figure 8.** FFT analysis of the modified model.



**Figure 9.** Stock without the additional output.