A Probability Model for TCP Slow Start

Asmir Gogic*, Nermin Suljanovic, Amer Hasanovic, Aljo Mujcic

[1] University of Tuzla, Faculty of Electrical Engineering Franjevacka 2, 75000 Tuzla, Bosnia and Herzegovina;
* *asmir.gogic@untz.ba*

Abstract.
This paper presents one approach to modeling of TCP connection during the slow start phase. Such modeling can be used for TCP connection analysis with reduced computation complexity compared to the packet-level simulators. Proposed model is validated by comparing the results obtained from ns-2 simulations.

Introduction

Application protocols mainly used on the Internet, such as HTTP or SMTP, use TCP protocol for reliable transport. TCP connection performance analysis can be carried out in two different ways. First one is simulation of TCP connection at the packet-level. This approach leads to accurate results but also requires long simulation time. Alternative approach is to model the TCP behaviour analytically, significantly reducing simulation time while simultaneously keeping accuracy at an acceptable level.

This paper reviews models available in the literature and proposes an alternative analytical TCP model during the slow-start phase. TCP behavior can be described with different models. Packet-level models are the most accurate since they employ full TCP stack implementation. However, when analyzing large-scale networks or simple networks with high throughputs, packet- level simulators are impractical due to long simulation time. An alternative approach is to use mathematical abstractions to model the TCP behavior. One way to abstract the TCP behavior through mathematical tools is to use differential equations. Another approach is based on probability analysis. In this approach, statistical formulas are used to describe TCP behaviour in different stages. Aggregating these models, the full TCP behavior could be obtained. In this paper, a probability model for the slow-start TCP stage is derived. The derived model is validated by comparing the results with the packet-level simulation tool ns-2 [7].

Finally, future directions for employment of probability models of other TCP stages are given.

1 TCP Protocol

TCP is a reliable connection-oriented transport protocol for packet-switched networks. Reliability is achieved by employing acknowledgements (ACKs) [2]. Using ACKs and sequence numbers, the transmitter keeps the track of packets that are successfully delivered to the receiver. TCP operates in different stages: Slow Start, Congestion Avoidance, Fast Retransmit, Fast Recovery and Timeout. Transition between stages is determined by packet loss or acknowledgement of predefined number of packets. The window size determines the maximum number of packets that a transmitter may send before receiving the first acknowledgement.

In the Slow Start phase, window size is incremented with every received ACK. Time interval, from departure of the first packet to the last packet in a window, represents round. The window size varies with the rate of the packet loss in the network. Hence, the packet loss probability increases with the number of sent packets due to the congestion in the network. Generally, window size w_i grows in rounds and can be expressed as:

$$w_i = w_{i-1} + \frac{1}{b} w_{i-1} = w_{i-1} \cdot \left(1 + \frac{1}{b}\right) = \gamma \cdot w_{i-1} = w_0 \cdot \gamma^{i-1} \tag{1}$$

where b is number of packets acknowledge by one ACK, and i is the round number and w_0 is the initial window size. Total number of packets sent in slow start including the round i is

$$ssdata_i = w_0 + \gamma \cdot w_0 + \gamma^2 + \cdots + \gamma^{i-1} \cdot w_0 = w_0 \cdot \frac{\gamma^i - 1}{\gamma - 1} \tag{2}$$

When a packet loss occurs during the slow start stage, there are two mechanisms to detect it. The first mechanism detects packet loss by using timeouts (TO). The second mechanism detects packet loss upon receiving three duplicate ACKs.

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2 Approaches for Modelling the Packet Switched Networks

There are several approaches for modeling packet - switched networks. The most accurate network models are *packet-level models* that keep track of individual packets. These models are implemented in network simulators, such as ns-2 [7]. The main drawback of packet-level model is a large computational effort required to keep track of each virtual packet in large scale simulations.

Analytical models are based on idea to model the TCP network mathematically in order to reduce complexity involved in simulations [1, 6]. These models do not consider the network dynamics. They assume that round-trip time and loss probability are constant and there are no interactions between TCP flows.

Fluid models are intended for simulation of packet networks. They overcome network scalability problem by keeping track of average quantities for relevant network parameters [5, 4]. Additionally, they use assumption that the bit rates are piecewise constant.

Hybrid models use continuous time state variables with discrete time events [3]. Hybrid simulations require significantly less computational resources than packet level simulators. However, solution of hybrid equations is still necessary in order to simulate networks.

3 Probability Model for TCP Slow-start Stage

There are two approaches for creating probability models of TCP behaviour. The source centric model assumes that packets leave the source with a certain loss probability [1]. The assumption taken by the second model is that the network generates loss probabilities for each packet [5]. Thus, the arrival process is represented by Poisson random process. In this paper we use the first approach with the further assumptions: the packet losses in two successive rounds are not correlated, packet loss occurs only in the forward direction; packet losses are independent of window size.

Model detects packet losses by triple duplicates and TOs. Based on the source-centric model [1,6], we derived the expected value of number of packets sent in the slow start phase as a function of packet loss probability p, the initial size of congestion window w_i and the number of packets acknowledged by one ACK. We further distinguish two boundary cases related to the position where packet was lost. The first case is characterized by the packet loss occurring at the beginning of the round (Figure 1). For the second case, the packet loss occurs at the end of the round (Figure 2).

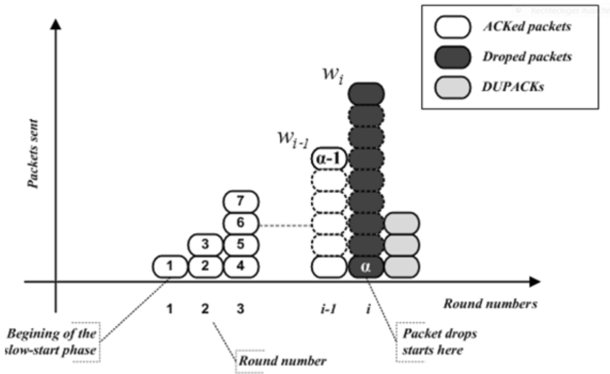

Figure 1. Slow-start phase, packet drop occurred at beginning of the round i.

Figure 1 shows the number of acknowledge packets α-1. Taking into consideration equation (2), we can write

$$ssdata_i = \alpha - 1 = w_0 \cdot \frac{\gamma^{i-1} - 1}{\gamma - 1} \qquad (3)$$

where α is a random variable. If the probability of packet loss is p, probability mass function *pmf* of having k-1 acknowledged packets is $(1-p)^{k-1}p$, the expected value of the discrete random variable α is

$$E[\alpha] = \sum_{k=1}^{\infty} (1-p)^{k-1} \cdot p \cdot k = \frac{1}{p} \qquad (4)$$

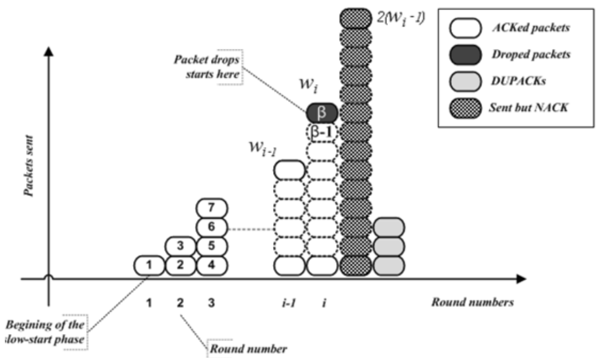

Figure 2. Slow-start phase, packet drop occurred at end of the round i.

Using the same principle for Figure 2, the number of acknowledge packets can be expressed as

$$ssdata_2 = \beta - 1 = w_0 \cdot \frac{\gamma^{i-1} - 1}{\gamma - 1} - 1 \qquad (5)$$

where β is random variable, and it is clear that $\beta > \alpha$ for the same round. Expected value of β is the same as expected value of α.

The exact number of acknowledged packets sent in slow start phase lies in the interval [$ssdata_1$, $ssdata_2$]. Width of this interval is $ssdata_2$ - $ssdata_1$ = β - α = w_i-1. Hence, in general case, the number of acknowledged packet in the slow start phase equals

$$ssdata_{ACK} = \alpha + w_i - 1 + \delta_i = w_0 \cdot \frac{\gamma^i - 1}{\gamma - 1} + w_i - \kappa_i \qquad (6)$$

where δ_i and κ_i are discrete random variables with values in the interval [0, w_{i-1}].

If we assume uniform distribution for these two variables, the expected values for these variables are equal and can be calculated as:

$$E[\kappa_i] = E[\delta_i] = \sum_{\delta_i=0}^{w_i-1} \delta_i \frac{1}{w_i} = \frac{w_i - 1}{2} \approx \frac{E[W] - 1}{2} \qquad (7)$$

where w_i is window size in round i. Since w_i is a random variable we are going to use its expected value

Using relations (4), (6) and (7) we can deduce expected value of window size:

$$E[W] = \frac{1-p}{p} \cdot (\gamma - 1) + w_0 \qquad (8)$$

The expected number of acknowledged packets in slow start phase, therefore, is:

$$E[ssdata_{ACK}] = \frac{1-p}{2 \cdot p} \cdot (\gamma + 1) + \frac{1}{2}(w_0 - 1) \qquad (9)$$

Using the same approach we can calculate the total number of packets sent in slow start phase as:

$$E[ssdata] = \frac{1-p}{2 \cdot p} \cdot (3 \cdot \gamma + 1) + \frac{1}{2}(3 \cdot w_0 - 1) \qquad (10)$$

4 Throughput in Slow Start Phase

Mean value of the throughput the receiver is experiencing during the slow start phase can be expressed as:

$$R = \frac{E[ssdata_{ACK}]}{E[i] \cdot t_{link} + E[ssdata_{ACK}] \cdot R_{link} + t_{ADT}} \qquad (11)$$

where $E[i]$ is expected round number value in which packet drop occurred, t_{link} is link delay, R_{link} is link speed t_{ADT} average packet loss detection time.

Probability of packet loss p_R occurring in round i is

$$p_R(i) = (1-p)^{w_0 \cdot \frac{\gamma^{i-1} - 1}{\gamma - 1}} \cdot p \cdot \sum_{k=0}^{w_0 \cdot \gamma^{i-1} - 1} (1-p)^k \qquad (12)$$

Hence, expected round number value $E[i]$ in which packet drop occurs is

$$E[i] = \lim_{N \to \infty} \sum_{i=1}^{N} i \cdot p_R(i) = \lim_{N \to \infty} \sum_{i=1}^{N} (1-p)^{w_0 \cdot \frac{\gamma^{i-1} - 1}{\gamma - 1}} \qquad (13)$$

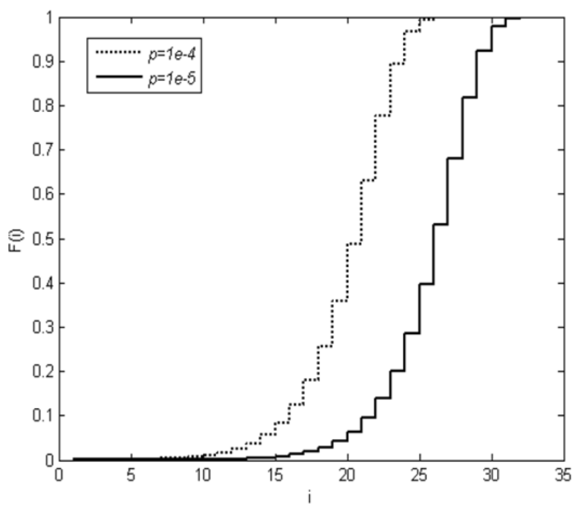

Figure 3. Cumulative distribution function for discrete random variable i.

For value, for which CDF shown in Figure 3 is higher than 0.9999995, the limit in equation (13) can be omitted and $E[i]$ can be calculated from the finite sum.

Average packet loss detection time interval t_{ADT} is mean time for detection of packet loss using timeouts or three duplicated ACK:

$$t_{ADT} = t_{TO} \cdot p_{TO} + t_{TD} \cdot p_{TD} \tag{14}$$

where p_{TO}, p_{TD} are probabilities of packet loss detection with timeout and three duplicated ACK, t_{TO} timeout interval and t_{TD} = RTT.

$$p_{TO} = p^{w_i} \cdot \left(1 + \frac{1-p}{p} \cdot (w_i - 1) + \frac{1}{2}\left(\frac{1-p}{p}\right)^2 \cdot (w_i - 1) \cdot (w_i - 1) \right) \tag{15}$$

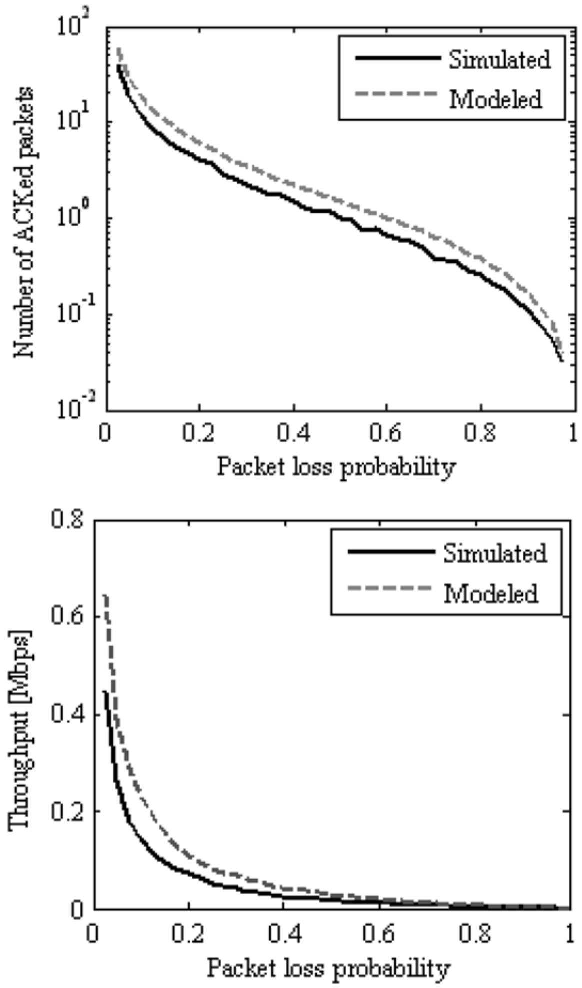

Figure 4. Simulation results: ns2 parameters: link speed 10Mbps, link delay 25ms, packet size 1024B, number of packet acknowledge by one ACK set to 1.

5 Simulation Results

Simulations were conducted in the packet level simulator ns2 [7], for a simple network topology containing only two nods. Figure 4 shows that the results obtained by probability analysis give appropriate upper boundary compared to the simulated results. Furthermore, it can be concluded that the error introduced by formula (9) grows as packet loss probability decreases. This leads to the conclusion that for lower probabilities more accurate model needs to be derived

6 Conclusion

In this paper, a probability model for the slow-start TCP stage is derived. The derived model is validated by comparing the results with the packet-level simulation tool ns-2. The test-case used for analyses was a simple to- pology consisting of two TCP nodes. Our analysis is confirmed by a large number of simulations. Similar ana- lysis should be performed for the other stages of TCP, and aggregated into a single model so that the full TCP behaviour could be analysed

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