Modelling of Heat Transfer Processes with Compartment/Population Balance Model

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A compartment/population balance model is presented for describing heat transfer in gas-solid fluidized bed heat exchangers, modelling the particle-particle and particle-surface heat transfers by collisions. The results of numerical experimentation, obtained by means of a second order moment equation model indicate that the model can be used efficiently for analysing fluidised bed heat exchangers recovering heat either by direct particle-fluid heat exchange or indirect tube-in-bed operation mode. The population balance model is validated with physically measured data taken from the literature [6].

Introduction

Fluidised bed heat exchangers, widely used in the metallurgical and process industries are important tools for recovering heat from hot solid particles [1-7]. In these systems heat exchange with the wall usually is modelled by means of suspension-wall heat transfer coefficients which, in principle, are aggregates of two transfer components: gas-wall and particle-wall heat transfers.

However, because of intensive motion of particles, the particle-wall, and also the particle-particle heat transfers are collision induced processes thus it seems to be significant to model these processes by themselves. Using such modelling approach the gas-wall and particle-wall components can be separated that allows understanding the transfer mechanisms involved.

For modelling and simulation of collisional heat transfer processes in gas-solid systems, an Eulerian-Lagrangian approach, with Lagrangian tracking for the particle phase [8-11], and a population balance approach [12-16] have been applied.

The population balance model, involving both the collisional particle-particle and particlewall heat transfers, was extended by Süle *et al.* [17, 18] for describing the spatial distributions of temperatures in deep or long fluidised beds developing a compartment model.

The aim of the paper is to extend the compartment population balance model to describe the heat transfer processes in fluidised bed heat exchangers in which the heat of hot solid particles is used to heat water flowing in tubes immersed in the bed. We apply a two-phase model of gas-solid fluidisation assuming that no bubbles are formed in the bed.

The particle-particle and particle-tube heat transfers are modelled by collisions, while the gasparticle, gas-tube and tube-water heat transfers are described as continuous processes using linear driving forces.

1 Physical model

Consider a shallow fluidised bed in which particles transported horizontally through the bed are fluidised by cross-flow air fed into the system in equally distributed gas streams along the bed. Cold water to be heated is flowing inside a tube immersed in the bed. The fluidising air induces intensive particle-particle and particle-tube collisions, and heat transfer between the gas, particles and water through the wall of the tube.

The assumptions concerning the system are as follows: 1) The particles are of constant size and are not changed during the process; 2) The system is operated under steady state hydrodynamic conditions, and the influence of thermal changes on the hydrodynamics is negligible. 3) There is no heat source inside the particles. 4) The heat transfer by radiation is negligible.

The structure of the compartments, as well as of the mass and heat flows of the system is shown in Figure 1. In this system the following mass transport processes are distinguished.

1. Volumetric cross-flow q_g of the fluidising gas through the ideally mixed cells between which some crossmixing occurs. The temperature of gas in the k-th cell is denoted by $T_{g,k}$, and there occurs continuous heat transfer between the gas and particles, and the gas and wall, characterised by the heat transfer coefficients h_{gp}, h_{gw}. and h_{wl}, respectively.
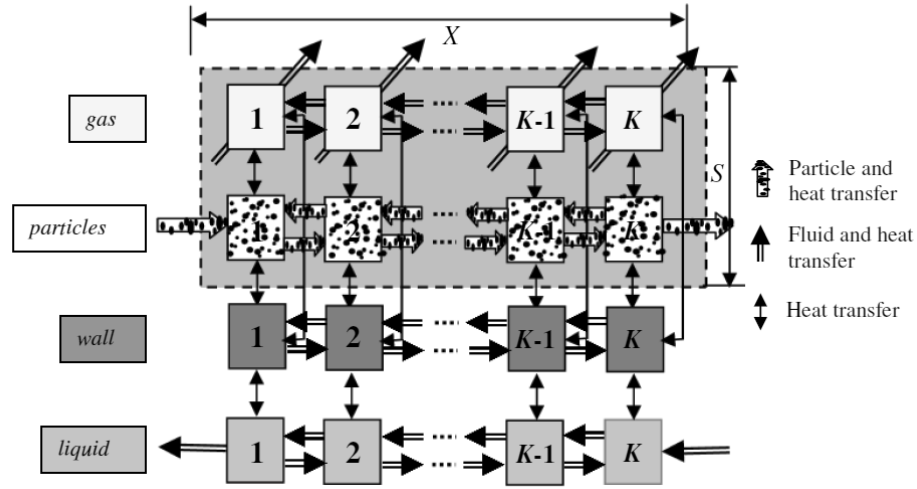
Figure 1 :the scheme of the system.

2. Dispersed plug flow of particles through the bed modelled by the cells-in-series with back-flow model. Here, $n_k(T_p, t)$ denote the population density function for the kth cell by means of which $n_k(T_p, t)\, dT_p$ provides the number of particles from the interval $(T_p,\ T_p + dT_p)$ in a unit volume of the cell at time t. Inter-particle heat transfer occurs by collisions, and is described by the random variable $\Omega_{pp} \in [0,1]$ with probability density function f_{pp}, while the particle-wall heat transfer also occurs by collisions that are characterised by the random variable with probability density function f_{pw}.

3. The heat in the wall of the tube is transported by conduction, and the continuous wall-liquid heat transfer is characterised by the heat transfer coefficient h_{wl}.

4. The volumetric flow q_l of water inside the tube is modelled also by the cells-in-series with back-flow model. It is assumed to be counter-current one with respect to the volumetric flow of particles.

In the present model, as it is illustrated in Figure 1, all compartments (cells) describing the shallow fluidised bed are of the same volume V_k, while, for the sake of computational simplicity, the number of discrete elements of the tube wall and of the cells of model of flowing liquid, although their volumes are quiet different, are the same as that of the bed compartments along the axial direction x.

2   Mathematical model

Under these conditions, the mathematical model of the heat transfer processes of the system is formed by a mixed set of partial integral-differential, partial differential and ordinary differential equations.

The population balance equation, which governs the variation of the population density function of particle population in the individual cells, is a partial integral-differential equation and can be written as

$$\frac{\partial}{\partial t} n_k(T_p, t) + \frac{a_{gp} h_{gp}}{c_p m_p} \frac{\partial}{\partial T_p} \left[(T_{g;k}(t) - T_p) n_k(T_p, t) \right]$$

$$= \frac{(1 + S_k R_p) q_p}{V_k} n_{k-1}(T_p, t) + \frac{S_{k+1} R_p q_p}{V_k} n_{k+1}(T_p, t)$$

$$- \frac{(1 + Z_k R_p) q_p}{V_k} n_k(T_p, t) - S_{pw} n_k(T_p, t)$$

$$+ S_{pw} \int_0^1 n_k \left(T_p - \frac{p_w \omega_{pw} T_{w;k}}{1 - p_w \omega_{pw}}, t \right) \frac{f_{pw}(\omega_{pw})}{1 - p_w \omega_{pw}} d\omega_{pw}$$

$$- S_{pp} n_k(T_p, t) + \frac{2 S_{pp}}{M_{0;k}} \int_{T_{p;min}}^{T_{p;max}} \int_0^1 \frac{f_{pp}(\omega_{pp})}{\omega_{pp}} \times$$

$$\times n_k \left(\frac{2(T_p - \tau)}{\omega_{pp}} + \tau, t \right) n_k(\tau, t)\, d\omega_{pp}\, d\tau$$

$$k = 1, 2, \dots, K, \ \tau > 0 \tag{1}$$

where the variables

$$\omega_{pp} := 1 - \exp\left[-\frac{2 h_{pp} a_{pp} \theta_{pp}}{m_p c_p} \right], \text{ and}$$

$$\omega_{pw} := 1 - \exp\left[-\frac{h_{pw} a_{pw} \theta_{pw} (m_p c_p m_w c_w)}{m_p c_p m_w c_w} \right] \tag{2}$$

represent the realizations of the random variables Ω_{pp} and Ω_{pw} which express, in principle, the efficiencies of the collisional particle-particle and particle-tube wall heat transfers [16].

Here, a_{gp}, a_{pp} and a_{pw} denote, respectively, the gas-particle, particle-particle and particle-wall contact area, while θ_{pp} and θ_{pw} denote the corresponding contact times. Parameter p_w expresses the ratio of thermal properties of particles and the wall, while parameters S_l and Z_l were introduced for characterising the compartmental structure of the system in a compact way where: $S_1 = 0$, $S_K = 1$, $Z_1 = Z_K = 1$, $S_l = 1, Z_l = 2, l = 2 \dots, K - 1$.

The first term on the left hand side of Eq.(1) denotes the rate of accumulation of particles having temperature $(T_p, T_p + dT_p)$, while the second term describes the change of the number of particles with temperature $(T_p, T_p + dT_p)$ due to the gas-particle heat transfer. The first three terms on the right hand side represent the input and output rates of particles from and to the neighbouring cells, as well as to and from the system, the next two terms describe the variation of the population density function due to the collisional particle-tube wall heat transfer, while the last two terms describe the change of $n_k(T_p, t)$ because of the collisional heat exchange between the particles.

The heat balance equations for the temperature of fluidising gas in the individual cells become

$$\frac{d}{dt}T_{g,1}(t) = \frac{q_{g,1}}{\varepsilon_1 V_1}T_{g,1,in}(t) - \frac{q_{g,1}}{\varepsilon_1 V_1}T_{g,1}(t) + \qquad (3)$$

$$+\frac{\varepsilon_2 R_g q}{\varepsilon_1 V_1}T_{g,2}(t) - \frac{\varepsilon_1(1+R_g)q}{\varepsilon_1 V_1}T_{g,1}(t) -$$

$$-\frac{a_{gw}h_{gw}}{c_g \rho_g}\int_0^{x_1}\left(T_{g,1}(t) - T_w(x,t)\right)dx -$$

$$-\frac{a_{gw}h_{gw}}{c_g \rho_g}\int_{T_{p,min}}^{T_{p,max}}\left(T_{g,1}(t) - T_p\right)n_1(T_p,t)dT_p$$

$$\frac{d}{dt}T_{g,k}(t) = \frac{q_{g,k}}{\varepsilon_k V_k}T_{g,k,in}(t) - \frac{q_{g,k}}{\varepsilon_k V_k}T_{g,k}(t) - \qquad (4)$$

$$-\frac{\varepsilon_{k-1}(1+R_g)q}{\varepsilon_k V_k}T_{g,k-1}(t) + \frac{\varepsilon_{k+1}R_g q}{\varepsilon_k V_k}T_{g,k+1}(t) -$$

$$-\frac{\varepsilon_k(1+2R_g)q}{\varepsilon_k V_k}T_{g,k}(t) -$$

$$-\frac{a_{gw}h_{gw}}{c_g \rho_g}\int_{x_{k-1}}^{x_k}\left(T_{g,k}(t) - T_w(x,t)\right)dx -$$

$$-\frac{a_{gw}h_{gw}}{c_g \rho_g}\int_{T_{p,min}}^{T_{p,max}}\left(T_{g,k}(t) - T_p\right)n_k(T_p,t)dT_p$$

$$\frac{d}{dt}T_{g,K}(t) = \frac{q_{g,K}}{\varepsilon_K V_K}T_{g,K,in}(t) - \frac{q_{g,K}}{\varepsilon_K V_K}T_{g,K}(t) - \qquad (5)$$

$$-\frac{\varepsilon_{K-1}(1+R_g)q}{\varepsilon_K V_K}T_{g,K-1}(t) - \frac{\varepsilon_K(1+R_g)q}{\varepsilon_K V_K}T_{g,K}(t) -$$

$$-\frac{a_{gw}h_{gw}}{c_g \rho_g}\int_{x_{K-1}}^{x_K}\left(T_{g,K}(t) - T_w(x,t)\right)dx -$$

$$-\frac{a_{gw}h_{gw}}{c_g \rho_g}\int_{T_{p,min}}^{T_{p,max}}\left(T_{g,K}(t) - T_p\right)n_K(T_p,t)dT_p$$

where ε_k denotes the bed voidage in the k-th cell, q stands for the volumetric gas flow between the cells causing some cross-mixing between the neighbour cells, and coefficient h_{gw} represents the gas-wall heat transfer rate.

Since the gas is assumed to be fed into the system equally distributed along the axial coordinate x we can write $\varepsilon_k = const.$ for all $k = 1, \dots, K$.

Heat in the wall of tube is transported with conduction hence the differential equation describing the temperature of the wall can be written in the form

$$\frac{\partial}{\partial t}T_w(x,t) = \frac{k_w}{\rho_w c_w}\frac{\partial^2}{\partial x^2}T_w +$$

$$+\frac{a_{gw}h_{gw}}{c_w \rho_w}\left(T_{g,k}(t) - T_w(x,t)\right) +$$

$$+\frac{a_{wl}h_{wl}}{c_w \rho_w}\left(T_w(x,t) - T_{l,k}(t)\right) +$$

$$+S_{pw}\int_{T_{p,min}}^{T_{p,max}}\int_0^1 p_p \omega_{pw}\left(T_p - T_w(x,t)\right)\times$$

$$\times\; n_k(T_p,t)f_{pw}(\omega_{pw})\,d\omega_{pw}\,dT_p$$

$$t > 0,\; x \in [(k-1)\Delta x, k\Delta x],\; k = 1, \dots, K \quad (6)$$

subject to the boundary conditions

$$\left.\frac{k_w}{\rho_w c_w}\frac{\partial}{\partial x}T_w(x,t)\right|_{x\in\{0,X\}} = 0 \qquad (7)$$

Here, k_w denotes the thermal conductivity of the wall, while the coefficient h_{wl} denotes the wall-liquid heat transfer rate. Parameters a_{gw} and a_{wl} denote the surface area of gas-wall and wall-liquid heat transfers in a unit length of tube.

Finally, the set of differential equations for the temperature of liquid phase compartments is written as

$$\frac{d}{dt}T_{l,k}(t) = \frac{S_{k-1}R_l q_l}{V_{l,k}}T_{l,k-1}(t) + \frac{(1+S_k R_l)q_l}{V_{l,k}}T_{l,k+1}(t)$$

$$-\frac{(1+Z_k R_l)q_l}{V_{l,k}}T_{l,k}(t) +$$

$$+\frac{a_{wl}h_{wl}}{c_l \rho_l}\int_{x_{k-1}}^{x_k}\left(T_w(x,t) - T_{l,k}(t)\right)dx \qquad (8)$$

where the values of the S and Z parameters, characterising the structure of the system are: $S_0 = S_K = 0$, $S_{K+1} = 0$, $Z_0 = Z_K = 1$, $S_l = 1$, $Z_l = 2$, $l = 1, \dots, K - 1$.

The additional boundary conditions to (1) – (8) can be written as

$$n_0(T_p, t) = n_{in}(T_p, t) \text{ and } T_{l,K+1} = T_{l,in} \qquad (9)$$

which, naturally, should be completed with the appropriate initial conditions.

3 Solutions of the model equation

The mixed set of differential equations (1)-(9) was solved by reducing the population balance equation (1) and the heat conduction equation (6) into two sets of ordinary differential equations applying, respectively, a moment equation reduction and a finite difference discretion, obtaining in this way together with the gas phase equations (3)-(5) and liquid phase equation (8) a closed set of ordinary differential equations.

The moments and normalized moments of the temperature of particle population are defined as

$$M_{I;k}(t) = \int_{T_{p,min}}^{T_{p,max}} T_p^I \, n_k(T_p, t) \, dT_p$$

$$m_{I;k}(t) = \frac{M_{I;k}(t)}{M_{0;k}(t)}; \ I = 0, \dots, k; \ k = 1, \dots, K \quad (10)$$

which are useful for the basic characterisation of the temperature distribution of particles. The zero order moments $M_{0;k}$ provide the total numbers of particles in a unit volume of cells by means of which the solids concentrations can also be computed, while the mean temperature of particles in the k-th cell is expressed by $m_{1;k}(t)$.

Completing the zero and first order moments $M_{0;k}$ and $M_{1;k}$ with the second order one $M_{2;k}$, the variance of temperature, defined as

$$\sigma_k^2 = \frac{M_{2;k}}{M_{0;k}} - \left(\frac{M_{1;k}}{M_{0;k}}\right)^2 \qquad (11)$$

can also be computed.

The infinite hierarchy of the moment equations generated by the population balance equation (1) has the form

$$\frac{d}{dt} M_{i;k} = IK_{gp} \left(M_{I-1;k}(t) T_{f;k}(t) - M_{i;k}(t) \right) +$$
$$S_{pp} \left[\frac{1}{M_{0;k}(t)} \sum_{i=0}^{I} M_{i;k}(t) \, M_{i-I;k}(t) b_{i,I}^{(1)} - M_{I;k}(t) \right]$$
$$+ S_{pw} p_1 \Big[-M_{I;k}(t) + \sum_{i=0}^{I} \binom{I}{i} \mathrm{b}_i^{(2)} \times$$
$$\times \sum_{j=0}^{i} \binom{i}{j} (-1)^{i-j} T_{w;k}^j M_{I-j;k}(t) \Big] +$$
$$+ \frac{(1+S_k R_p) q_p}{V} M_{I;k-1}(t) + \frac{R_p q_p}{V} M_{I;k+1}(t) -$$
$$- \frac{(1+Z_k R_p) q_p}{V} M_{I;k} \qquad I = 0,1, \dots \qquad (12)$$

where

$$b_{i,I}^{(pp)} = \int_0^1 \binom{I}{i} \left(\frac{\omega_{pp}}{2}\right)^2 \left(1 - \frac{\omega_{pp}}{2}\right)^{I-i} f_{pp}(\omega_{pp}) \, d\omega_{pp}$$

$$\text{and } b_i^{(pw)} = \int_0^1 \omega_{pw}^i f_{pw}(\omega_{pw}) \, d\omega_{pw} \qquad (13)$$

Since the infinite set of moment equations can be closed at any order, the second order moment equation reduction can be computed exactly by solving the equations for the first three leading moments. This reduction is obtained by using the following equations.

The total number of particles in the k-th cell:

$$\frac{d}{dt} M_{0;k}(t) = \frac{(1+S_k R_p) q_p}{V} M_{0;in}(t) + \frac{R_p q_p}{V} M_{0;k+1}(t)$$
$$- \frac{(1+Z_k R_p) q_p}{V} M_{0;k}(t) \quad k = 1, \dots, K \qquad (14)$$

The first order moment of the particulate phase in the k-th cell:

$$\frac{d}{dt} M_{1;k}(t) = \frac{a_{gp} h_{gp}}{c_g \rho_g} \left(M_{0;k}(t) T_{g;k}(t) - M_{1;k}(t) \right)$$
$$+ S_{pw} p_w b_1^{(2)} \left(M_{0;k}(t) T_{w;k}(t) - M_{1;k}(t) \right) +$$
$$+ \frac{(1+S_k R_p) q_p}{V} M_{1;in}(t) + \frac{R_p q_p}{V} M_{1;k+1}(t) -$$
$$- \frac{(1+Z_k R_p) q_p}{V} M_{1;k}(t), \quad k = 1, \dots, K \qquad (15)$$

The variance of temperature of the particulate phase in the k-th cell:

$$\frac{d}{dt} \sigma_k^2(t) = - \Big[2 \frac{a_{gp} h_{gp}}{c_g \rho_g} + S_{pp} b_{1,2}^{(pp)} + S_{pw} p_w \left(2 b_1^{(pw)} - \right.$$
$$- p_w b_2^{(pw)} \Big) + \frac{q_p}{V M_{0;k}(t)} \left(R_p M_{0;k+1}(t) \right) \Big] \sigma_k^2(t) +$$
$$+ \frac{q_p}{V} \frac{(1+S_k R_p) M_{0;in}(t)}{M_{0;k}(t)} \left(\sigma_{in}^2(t) - \sigma_k^2(t) \right) +$$
$$+ S_{pw} b_2^{(pw)} p_w^2 \left(\frac{M_{1;k}(t)}{M_{0;k}(t)} - T_{(w;k)}(t) \right)^2 +$$
$$+ \frac{q_p (1+S_k R_p) M_{0;in}(t)}{V M_{0;k}(t)} \left(\frac{M_{1;in}(t)}{M_{0;in}(t)} - \frac{M_{1;k}(t)}{M_{0;k}(t)} \right)^2 +$$
$$+ \frac{R_p q_p M_{0;k+1}(t)}{V M_{0;k}(t)} \sigma_{k+1}^2(t) +$$
$$+ \frac{R_p q_p M_{0;k+1}(t)}{V M_{0;k}(t)} \left(\frac{M_{1;k+1}(t)}{M_{0;k+1}(t)} - \frac{M_{1;k}(t)}{M_{0;k}(t)} \right)^2 \qquad (15)$$

The set of equations provided with finite difference discretion of the heat conduction equation (6) for the wall has the form:

$$\frac{d}{dt}T_{w,1}(t) = \frac{D_T}{\Delta x^2}\left(T_{w,2}(t) - T_{w,1}(t)\right) -$$

$$\frac{a_{gw}h_{gw}}{c_w\rho_w}\left(T_{g,1} - T_{w,1}\right) - \frac{a_{wl}h_{wl}}{c_w\rho_w}\left(T_{w,1}(t) - T_{l,1}(t)\right)$$

$$-S_{pw}\int_{T_{p,min}}^{T_{p,max}}\int_0^1 p_p\left(T_{w,1}(t) - T_p\right)n_1(T_p,t) \times$$

$$\times \omega_{pw}f_{pw}(\omega_{pw})\,d\omega_{pw}\,dT_p \qquad (17)$$

$$\frac{d}{dt}T_{w,k}(t) = \frac{D_T}{\Delta x^2}\left(T_{w,k+1}(t) - 2T_{w,k} + T_{w,k-1}\right) +$$

$$+\frac{a_{gw}h_{gw}}{c_w\rho_w}\left(T_{g,k}(t) - T_{w,k}(t)\right) - \frac{a_{wl}h_{wl}}{c_w\rho_w}\left(T_{w,k}(t) - T_{l,k}(t)\right)$$

$$-S_{pw}\int_{T_{p,min}}^{T_{p,max}}\int_0^1 p_p\left(T_{w,k}(t) - T_p\right)n_k(T_p,t) \times$$

$$\times \omega_{pw}f_{pw}(\omega_{pw})\,d\omega_{pw}\,dT_p \qquad (18)$$

$$\frac{d}{dt}T_{w,k}(t) = \frac{D_T}{\Delta x^2}\left(T_{w,K}(t) - T_{w,K-1}(t)\right) +$$

$$+\frac{a_{gw}h_{gw}}{c_w\rho_w}\left(T_{g,K}(t) - T_{w,K}(t)\right) - \frac{a_{wl}h_{wl}}{c_w\rho_w}\left(T_{w,K}(t) - T_{l,K}(t)\right)$$

$$-S_{pw}\int_{T_{p,min}}^{T_{p,max}}\int_0^1 p_p\left(T_{w,K}(t) - T_p\right)n_K(T_p,t) \times$$

$$\times \omega_{pw}f_{pw}(\omega_{pw})\,d\omega_{pw}\,dT_p \qquad (19)$$

so that the integrals of variable x in $(3-5)$ for the fluidising gas and (8) for the liquid flowing in the tube are also rewritten for the discrete values $T_{w,k}$, $k = 1, 2, …, K$.

Finally, because of the second order moment equations reduction $(14-16)$, in $(3-5)$ for the fluidising gas and $(17-19)$ for the wall in finite difference form the integrals of variable T_p have to be changed rewriting these integrals applying the moments of the temperature of particle population:

$$\int_{T_{p,min}}^{T_{p,max}}\frac{a_{gp}h_{gp}}{c_g\rho_g}\left(T_{g,k}(t) - T_p\right)n_k(T_p,t)\,dT_p =$$

$$= \frac{a_{gp}h_{gp}}{c_g\rho_g}\left(M_0(t)T_{g;k}(t) - M_{1;k}(t)\right) \qquad (20)$$

and

$$\int_{T_{p,min}}^{T_{p,max}}\int_0^1 p_p\left(T_{w,K} - T_p\right)n_K\,\omega_{pw}f_{pw}\,d\omega_{pw}\,dT_p$$

$$= p_p b_1^{(2)}\left(M_0(t)T_{w;k}(t) - M_{1;k}(t)\right) \qquad (21)$$

4 Simulation results and discussion of the model

A computer program was developed in MATLAB environment for solving the set of ordinary differential equations $(3-5)$, (8) and $(14-19)$ taking into account all modifications of the integral terms. The program can generate and handle a compartment/moment equations model consisted of cells of arbitrary number, and the resulted set of ordinary

differential equations is solved by means of an ode-solver of MATLAB.

The transient and steady state simulation results presented here were obtained for 9 cells using the basic constitutive expressions presented in detail in [16].

The predictions of the second order moment equation reduction model were validated using the experimental data measured in a laboratory shallow fluidized bed heat exchanger published by Pécora and Parise [6].
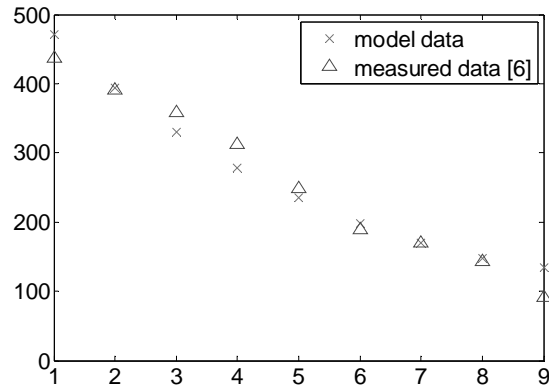
Figure 2. The results of the compartment/population balance model compared with the experimental data by Pécora and Parise [6]

Figures 2 and 3 present the bed temperature profiles for 9 cells comparing the model data with the measured ones [6] when the input temperatures of particles were 650 °C and 708 °C, respectively. The parameters were fitted to the measured values using a least squares method. The results in both cases show rather good correspondence but it has to be taken only as preliminary ones since the heat transfer coefficients have been compared yet.
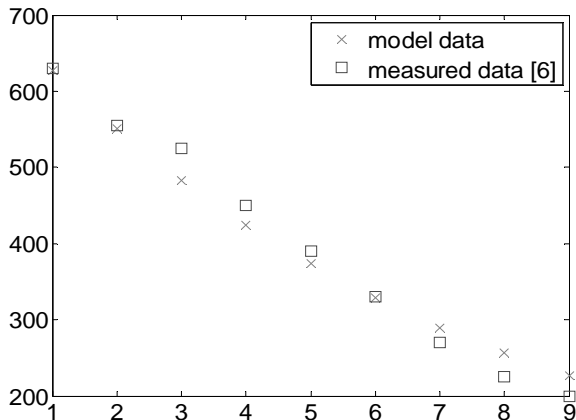
Figure 3. The results of the compartment/population balance model compared with the experimental data by Pécora and Parise [6]
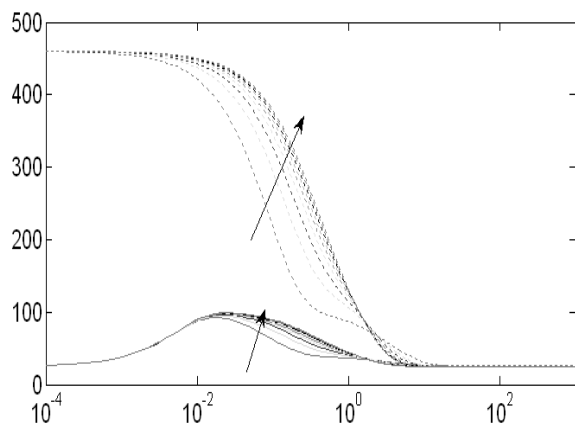
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Figure 4. Transients of the mean temperature of particles and the temperature of air in the cells
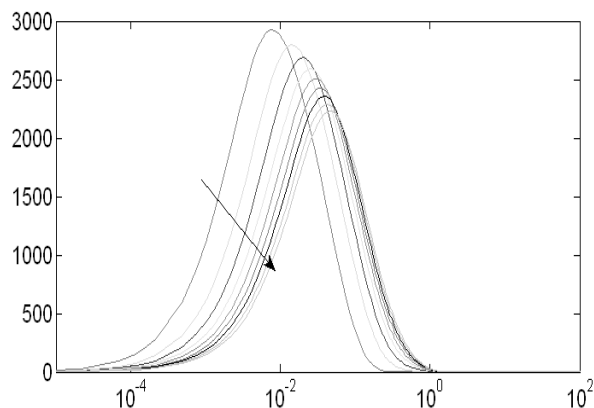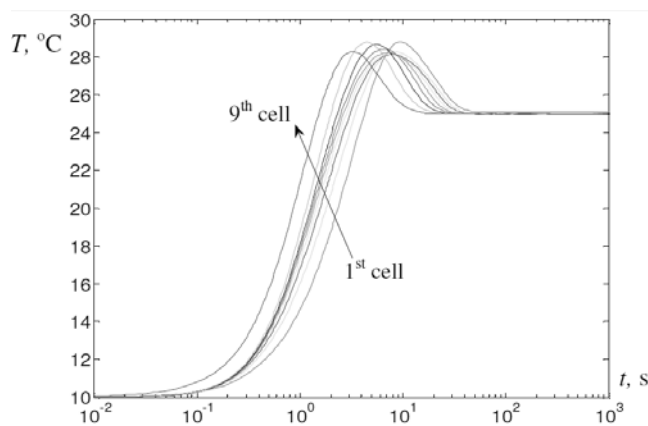

Figure 5. Transients of the temperature of water flowing in the tube

Figure 6. Transients of the variance of temperature of particle population.

The simulation results have shown that the gas-particle and particle-wall heat transfers induce inhomogeneities of the temperature of particles but the particle-particle collision heat transfer shows a strong indirect effect.

5 Conclusions

The compartment/population balance model, developed for describing heat transfer processes in gas-solid fluidized bed heat exchangers, and modelling the particle-particle and particle-surface heat transfer processes by collisions can be used efficiently for analysing the fluidized bed heat exchangers recovering heat from hot particles and heating some liquid flowing in a tube immersed in the bed.

The model describes the temperature distribution of the particle population, and allows separating the effects of the fluidizing gas-immersed surface and particle-immersed surface heat transfers.

The second order moment equation reduction, generated from the population balance equation has proved to be an efficient tool for studying the behaviour of heat exchangers.

Figure 4 presents the transients of the mean temperature of particles and the temperature of the fluidizing air in the cells along the heat exchanger. It is seen that in steady state these temperatures become almost equal and the heat of hot particles becomes transferred to the cold water.

Under such conditions, the temperature of gas passes a maximum in each cell but delayed to each other in time. Similar maxima can be observed also in the transient processes of the wall and in the temperature of liquid, as it is presented in Figure 5, heated by the hot particles through the tube wall.

Figure 6 shows the variation of the variance of temperature of particle population as a function of time. The temperature of particles at the input was homogeneous but it became strongly distributed during the transient process showing rather large variances.

	Parameter	Basic value
Solid particles: sand	Diameter, d_n	2.54×10^{-4} m
	Density, ρ	2650 kg m^{-3}
	Specific heat, c_n	835 J kg^{-1} K^{-1}
	Thermal conductivity, k_n	0.35 W m^{-1} K^{-1}
	Volumetric flow rate, a_n	1.5×10^{-5} m^3 s^{-1}
	Mean inlet temperature, m	460
Gas: air	Density, ρ	0.946 kg m^{-3}
	Specific heat, c_a	1010 J kg^{-1} K^{-1}
	Viscosity, μ	2.17×10^{-5} Pa s
	Thermal conductivity, k_a	2.39×10^{-2} W m^{-1} K-1
	Volumetric flow rate, a_a	1.4×10^{-2} m^3 s^{-1}
	Inlet temperature, $°C$	25
Tube wall: stainless steel	Diameter, D_l	0.0065 m
	Mass, m_w	1.2 kg
	Specific heat, c_w	465 J kg^{-1} K^{-1}
	Thermal conductivity, k_w	44 W m^{-1} K^{-1}
Heated medium: water	Density, ρ	998 kg m^{-3}
	Specific heat, c_l	4182 J kg^{-1} K^{-1}
	Thermal conductivity, k_l	0.606 W m^{-1} K^{-1}
	Viscosity, μ	10-3 Pa s
	Volumetric flow rate, a_l	1.5×10-5 m^3 s^{-1}
	Inlet temperature, $°C$	25
Fluidized bed	Width, W	0.15 m
	Length, L	0.9 m
	Collision freq., S_{nn}, S_{nw}	103 s^{-1}, 10 s^{-1}
	Heat transfer eff. u_{nn}, u_{nw}	0.5, 0.8
Back flow ratio	R_n, R_a, R_l	1, 0.1, 0.01

Table 1. The basic constitutive and process parameters used in simulation

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Table of Symbols

a	surface area, m^2
c	specific heat, J kg^{-1} K^{-1}
W	width of the bed, m
D	diameter of the tube, m
D_T	thermal diffusivity, $D_T = k_w/(\rho_w c_w)$, m^2 s^{-1}
f	probability density function
h	heat transfer coefficient, W m^{-2} K^{-1}
k	thermal conductivity, W m^{-1} K^{-1}
m	mass, kg
M_k	k-th order moment of particle temperature
m_k	normalised k-th order moment of particle temperature
n	population density function, no m^{-3} K^{-1}
p_p	parameter in Eq.(1), $p_p = \frac{m_w c_w}{m_p c_p + m_w c_w}$

p_w	parameter in Eq.(1), $p_w = \frac{m_p c_p}{m_p c_p + m_w c_w}$
q	volumetric flow rate, m^3 s^{-1}
R	back-flow ratio
T	temperature, K
t	time, s
V	volume, m^3
x	axial variable, m
θ	contact time, s
ω	random variable of collision heat transfer
μ	viscosity, Pa s
ε	void fraction of the bed
σ^2	variance of the temperature of particle population
ρ	density, kg m^3

Table of subscripts and superscripts

g	gas
in	input
l	liquid
max	maximal value
mf	incipient velocity of fluidisation
min	minimal value
p	particle
gp	gas-particle
pp	particle-particle
pw	particle-wall
w	wall
gw	gas-wall
wl	wall-liquid

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